Gravitational Energy and Energy Conservation in General Relativity and Other Theories of Gravity

DPhil Thesis

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Content

I. Introduction

Armed with the tools for metaphysical, methodological and conceptual-logical in-depth analyses of specific theories, philosophy of physics seeks to deepen our understanding of modern physics, and aspires to offer tentative heuristic guidance to scientists (see e.g. Kanitscheider, 2009).

This dissertation will centre upon a particularly knotty conundrum within philosophy of physics: the re-conceptualisation of gravity as a manifestation of the curved geometry of spacetime, inaugurated by Einstein's theory of General Relativity (GR). Whereas in standard Newtonian theory gravity is conceived of as a force deflecting particles from their rectilinear paths of inertial/force-free motion, according to GR's geometric interpretation, they trace out – like (frictionless) snowboarders gliding in a half-pipe – the shortest paths of a non-Euclidean geometry of spacetime (the fusion of space and time, no longer immutable or mutually independent).

Notwithstanding GR's indubitable success as a physical theory, the "geometrisation" of gravitational physics that it's supposed to instantiate, poses a number of formidable philosophical challenges. One is to articulate the precise sense in which theories can be said to geometrise physics. This may be perceived as a particularly pressing task not only for systematic reasons (epitomised in the rich historical literature on unified field theories modelled on, or inspired by GR's geometrisation, see e.g. Vizgin, 2011; Goenner, 2004). Ironically, Einstein himself disavowed GR's geometric interpretation – an interpretation that is nowadays considered orthodoxy (attested to, for instance, by Misner, Thorne & Wheeler, 1973). For Einstein, geometrising gravity not only was inessential to GR; he even disputed that it has a substantive meaning, at all (see Lehmkuhl, 2014 for details).[1]

Once different senses of geometrisation are forthcoming, another question immediately ensues: to what extent do (or can) they be applied to GR (and other theories)? In our endeavour to deepen our understanding of GR and its peculiarities, we must be wary of the pitfalls of gravity/GR-"exceptionalist" (Pitts, 2016ab, 2017, 2019) double standards. Therefore, also a comparison with other theories – both gravitational, e.g. Brans-Dicke Theory,

[1] The emphasis on geometisation of (gravitational) physics was championed in particular by Weyl and Eddington (see e.g. Ryckman, 2005, Ch. 8-9; 2017, sect. 5)

and non-gravitational, e.g. Yang-Mills theories of contemporary high-energy physics – is apposite: to what extent may they, too, be legitimately classified as geometric theories?

Finally, these questions pave the way for a normative question: *should* we adopt GR's geometric interpretation? What are its advantages vis-à-vis, say, Einstein's own unificatory interpretation (Lehmkuhl, 2014), or Feynman's (1995) (and others') spin-2 graviton view (cf. Salimkhani, 2020)?

In his seminal work, Lehmkuhl (2009a; 2009b, Ch. IX; 2014; cf. Goenner, 2008) has addressed the first set of questions – concerning the explication of geometrisation. He distinguishes between three kinds (or "strengths"):

- **Strength-1** geometrisation dresses up field theories in geometric clothing. The fiber bundle formulation of electromagnetism is a case in point: while in such a *representation* everything looks geometric, we have prima facie little reason to regard the theory as describing anything inherently related to spacetime geometry.
- In **strength-2** geometrisation, physical degrees of freedom can be accounted for in terms of geometric properties (e.g. topology, curvature, or torsion) of *augmented* spacetime structure.[2] An example is Weyl's (1918, 1919) unified field theory. In it, the electromagnetic field is re-conceptualised ("strength-2 geometrised") as a manifestation of what Weyl called "length curvature" of a non-Riemannian spacetime (i.e. a spacetime in whose geometry parallel transport of vectors alters their length).
- **Strength-3** geometrisation, paradigmatically instantiated by GR's geometric interpretation, is essentially *eliminative*: a geometric theory of strength-3 reduces physical degrees of freedom to manifestations of (a universal) inertial structure[3] – a preferred path structure of "natural", uncaused/default motion that, for instance, force-force test particles trace out.

[2] One may wonder to what extent the gauge theory framework, couched in the geometric language of fibre bundles (see e.g. Naber, 2011, Ch. 4-6), falls under strength-2 geometrisation: for the relevant "enriched" geometries, to each point an *internal* space (representing the internal degrees of freedom, such as, say, isospin or colour) is attached. Such a thesis – under the apt label "geometrodynamics of gauge fields" – has indeed been advocated by Mielke (1987).
In this thesis, however, I won't further pursue this intriguing question. Spacetime geometry will be treated throughout conservatively – as a geometry of *external* degrees of freedom (eliciting universal effects).

[3] I'll gloss over details of this reduction to inertial structure – to be taken up in future work. In this regard, one question in particular seems important. It's related to the fact that GR, in contrast to theories such as those envisioned by Eddington (1923, Ch. VI, Part II) or Schrödinger (1950, Ch. 12) isn't a *purely* affine theory. More precisely, the question is: to what extent does this inertia-reductive interpretation do full justice to GR's actual physical content? Strictly speaking, inertial structure is given by affine structure, represented by GR's Levi-Civita connection. But GR is *also* – if not, first and foremost! – about metric structure (which, of course, determines the affine structure, due to the metric compatibility of the assumed underlying Riemannian geometry), containing

Drawing on Lehmkuhl's classification, the subsequent work will hone in on the ramifications of the different strengths of geometrisation: what follows for a geometric theory's ontology and ideology (in the sense of Quine, 1951)? That is: what can meaningfully be said about the world within this theory's conceptual resources? Primarily, I'll be concerned with strength-3 geometrisation – desirable, qua parsimony, for its *unificatory power* (cf. Kitcher, 1981, 1989): taking a "strength-3 geometric" theory at face value, what propositions about the world would be true, if this theory were to be accepted?

I'll focus on energy and energy conservation – two notions integral to virtually all of physics, and usually deemed fundamental (cf. Hiebert, 1962; Lindsay, 1971; Elkana, 1974; Harman, 1982 for a historical perspective on their centrality).

In modern spacetime theories, their status, however, can't be taken for granted anymore. Consider, for instance, the following tentative suggestions:

- In Analytic Mechanics, conservation of energy and momentum are usually presented as associated with the homogeneity of time and space, respectively (see e.g. Landau & Lifshitz, 1976, Ch. 2). But if spacetime theories typically allow for space and time to be warped, shouldn't we expect energy-momentum conservation to forfeit its unqualified validity?
- Norton (2003, p. 20) writes: "The first thing that one learns in approaching general relativity is that the notion of force has been compromised. […] It has been 'geometrised away'. But one cannot geometrise away force without other ramifications. If gravitational force has somehow been compromised – geometrised away – then we should expect the same to happen to the other dynamical quantities in [the equations 'ENERGY = FORCE x SPACE' and 'MOMENTUM= FORCE x TIME']." Shouldn't we likewise expect gravitational energy to be "geometrised away"?

The working hypothesis that will guide the following analyses is thus: modern spacetime theories significantly affect the status of both energy and its conservation. The arguments proffered will turn both on conceptual analysis of various theories, as well as on explanatory practices.

More specifically, I'll moot two claims (to be fleshed out in greater detail in due course):

chronogeometric information (i.e. information about lengths, durations, volumes, and the causal/light-cone/causal structure). Metric structure is indeed indispensable: the electromagnetic energy-stress tensor, for instance, requires a metric for its very definition; the Levi-Civita connection by itself *doesn't* suffice (cf. Lehmkuhl, 2011 for a more general elaboration of this thought). In light of this, it would seem more accurate to consider GR a theory that reduces gravity to chronogeometric-*cum*-inertial structure.

> **(C1)** Geometrisation can force on us revisions of the status of the energy-momentum associated with the geometrised quantity (i.e. in the cases considered: gravity). In strength-3 geometric theories of gravity, such as GR, in particular, gravitational energy becomes problematic: I'll argue for eliminativism about it.
>
> **(C2)** Energy conservation is no longer a fundamental, apodictic principle. It's contingent on whether the spacetime possesses appropriate symmetries. (Hence, the status of energy conservation is orthogonal to geometrisation per se.) For GR, in particular, energy-conservation is violated (except in the absence of gravity).

Such conclusions may appear radical – as perhaps befits a revolutionary theory like GR. Given the latter's revolutionary character, we are well-advised to strive for its deeper understanding, especially of all of its consequences. With regard to radical revisions of our received conceptual toolkit, GR – or modern spacetime theories, more generally – doesn't stand alone: quantum field theory and (a fortiori) semi-classical gravity (i.e. quantum field theory on curved spacetime) likewise face difficulties in even defining energy and its conservation (Kiefer, 2007, Ch. 1.2; Maudlin, Okon & Sudarsky, 2019).

The present study has relevance for physics, philosophy of physics and metaphysics. It enhances our understanding of the deep link between spacetime structure and laws of nature in both GR and the most significant alternative theories of gravity (both historical and contemporary) – a topic both of intrinsic interest to metaphysics of physics (e.g. Friedman, 1984, 2001; DiSalle, 2006; Belot, 2011; Lange, 2013; Sus, 2019), and furthermore at the heart of a major debate within philosophy of physics – that between advocates of so-called dynamical and geometric approach to spacetime, respectively (e.g. Brown, 2005; Brown & Read, 2019; Read 2019; Read, 2020; Weatherall, 2020).

Specifically, by defending the view that on GR's standard geometric interpretation, neither gravitational energy exists in GR – i.e. (C1) –nor that energy conservation holds without qualifications – i.e. (C2) – I'll present a coherent solution to a long-standing dilemma in the GR literature: Gravitational energy seems to defy all attempts to localise it – that is, to specify in which region of spacetime it resides. Whereas orthodoxy (e.g. Misner, Thorne & Wheeler, 1973, §20.4) deems this a brute fact one needs to swallow, the proposed solution, (C1) & (C2), offers an eliminative explanation of this fact (analogous to Lavoisier's explanation of combustion by elimination of phlogiston).

This Gordian solution is intimately intertwined with black hole thermodynamics – a hot topic of current research, targeting a potentially fundamental link between thermodynamics and (general-relativistic) gravity (see e.g. Wallace, 2017ab; Curiel, 2019, sect.5). Gravitational energy features explicitly in the First Law, which states an energy conservation law via a relation between the black hole‘s area, angular momentum and electric charge. But to what extent the analogy between ordinary thermodynamics and the laws of black hole thermodynamics is substantive, remains controversial (Dougherty & Callender, 2019). The existence of gravitational energy and the validity of energy conservation in GR prima facie have a crucial bearing on this assessment.[4]

The present project should attract also the interest of metaphysicians. Carrying energy is frequently cited (e.g. by Bunge, 2000 or Norton, 2003, p. 18) as a (sufficient) criterion for the reality of a physical entity. Thus, whether we can meaningfully assign energy-momentum to the metric – plausibly viewed as both encoding the gravitational degrees of freedom and endowed with spatiotemporal significance – plays a pivotal role in the debate over the substantival, "thing-like" nature of spacetime (as opposed to the view that it's a fictitious abstraction into which we embed (for reasons of expedience) relational, spatiotemporal facts about material objects, see Earman, 1989; Friedman, 1983, Ch. VI for reviews). For instance, for Earman and Norton (1987, p. 519) the very categorical difference between substantival spacetime ("container") and matter ("the content of spacetime") hinges on energy: "If we do not classify such energy bearing structures [...] as contained within space-time, then we do not see how we can consistently divide between container and contained." Eliminativism about gravitational energy raises the question (cf. Hoefer, 2000) whether carrying energy is a sufficient or also a necessary criterion for an entity's reality. What could be alternative criteria?

This is closely related to causal efficacy: on a widespread conception of causality (e.g. Salmon, 1998; Dowe, 2000), transmission of energy-momentum is regarded as the essence of causation.[5] Hence, on such models, spacetime structure – or more precisely, certain types of

[4] A promising conjecture, *compatible* with both (C1) and (C2) in the context of GR, and, at the same time, an affirmative stance towards black hole thermodynamics, is that in the regime of black holes GR should be seen as merely an effective field theory description (cf. Crowther, 2016), not a fundamentally accurate description. This in turn engenders interesting questions regarding inter-theory relations, exhibited in the case of black hole thermodynamics.

[5] In this context, some authors have also seen a connection between the topics covered in this thesis and philosophy of mind – Leibniz's question of how human minds/souls, construed as dualistic substances, could

geometrised quantities, affected by (C1) – doesn't qualify as causally efficacious. What about alternative models of causation, though – models that don't rely on the transmission of energy-momentum (such as counterfactual theories, e.g. Reutlinger, 2016ab, 2017, 2018; French & Saatsi, 2018)? The denial of energy-momentum conservation and the existence of gravitational energy in certain gravitational theories thus invites case studies for such models of causation (cf. Hoefer, 2009, sect. 4), or even anti-realism about causality in the spirit of Russell (e.g. Norton, 2007; Ladyman & Ross, 2007, Ch. 5), i.e. the view that one should reject causality as a fundamental element of reality: to what extent do they apply to GR and other spacetime theories?

This in turn feeds back into philosophy of physics: how should we construe the interdependence between spacetime structure and the matter degrees of freedom in strength-3 geometric theories of gravity, as encapsulated in the Einstein Equations (or their counterparts) – for instance, as a *causal* (in some metaphysically thick sense) interaction (as suggested by Wheeler, Misner and Thorne's (1973, p. 5) famous slogan that "space tells matter how to move", and conversely "matter tells space how to curve"), or merely as a nomological mutual constraint (see e.g. Nerlich, 2013, Ch. 8-9; Vasallo, 2019)?[6]

Those exciting questions will have to be tackled in future research. The task for now is to study geometrisation of general-relativistic gravity and its ontological and ideological implications. This will be the unifying theme of the subsequent chapters – five case-studies of various theories of gravity, with special emphasis on GR – comprising this thesis. Finally, a disclaimer: throughout, I won't question GR's geometric interpretation, whose ramifications I intend to unpack (cf. however Dürr, ms). This premise is for purely systematic reasons: I deny neither the existence nor the merits of other interpretations.

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interact with matter: at first blush, such an interaction would conflict with energy conservation (see Cucu & Pitts, 2019; Pitts, 2019abc, 2020 for details). Given energy conservation, the mind/body dualism therefore, the argument concludes, becomes problematic.

I deem such an emphasis on energy conservation largely a red herring. The real problem for an interactionist mind-body dualist (or an advocate of divine or "spiritual" intervention) is more general. Suppose that one commits to the causal/nomological closure of the material/physical world, broadly construed, i.e. the view that reality is structured by uniform regularities, captured satisfactorily by laws of nature. Then, how to square this view with the influence of immaterial entities upon this world? Whether a particular scientific theories abandons energy conservation is irrelevant, as long as this causal closure remains intact: then, the laws of nature leave "no room" for mental/spiritual (or divine) agency. The failure of energy conservation for which I'll argue satisfies this priviso: it asserts a quantifiable non-conservation – the extent of which is specified by GR.

[6] This could shed light on how to understand the action-reaction principle and its epistemological and metaphysical status (cf. Brown & Lehmkuhl, 2013).

The thesis is structured as follows:

Chapter II will critically examine the standard arguments, nigh-universally cited in the literature that are supposed to demonstrate that gravitational waves carry energy-momentum. These standard arguments are taken to establish that gravitational energy *must* exist in GR.

In **Chapter III**, I'll analyse the status of energy conservation in GR, and discuss various candidates for local (differential) notions of gravitational energy. Special emphasis is given to pseudotensors, and Pitts' recent proposal.

Chapter IV extends the analysis of the previous chapter to global (integral) notions of gravitational energy. Special attention is given to the status of asymptotic flatness as an idealisation. By critically evaluating a recent proposal by Read, I perspicuously state the challenges that realism about gravitational energy in GR faces.

In **Chapter V**, I turn to Newtonian Gravity. I consider the status of gravitational energy in various formulations (of different strengths of geometrisation), with special emphasis on Newton-Cartan theory – a strength-3 geometrised variant of Newtonian Gravity.

Chapter VI is a historical-critical study of Nordström Gravity, a precursor of GR. I present a more circumspect formulation of it that provides a unified – and as it turns out, strength-3 geometric – description of both of its historical variants. This allows a clarification of the status of the Geodesic Principle, energy conservation and gravitational energy in Nordström Gravity.

In **Chapter VII**, I summarise and evaluate the results of this dissertation. Lines of future inquiry are sketched.

The next chapter:

Why assume that gravity should be ascribed energy? To be sure: An abnormally strong eruption of gravitational waves could tragically destroy the Radcliffe Camera or the Cathedral of Learning! But is that enough to warrant the claim? We'll next critically examine the standard arguments for the belief that gravitational radiation in General Relativity *"of course"* carries energy.

II. Gravitational Waves and Energy Transport

Abstract:

In this chapter, I review and critically examine the four textbook arguments commonly taken to establish that gravitational waves (GWs) carry energy-momentum: 1. the increase in kinetic energy that a GW confers on a ring of test particles, 2.Bondi/Feynman's Sticky Bead Argument of a GW heating up a detector, 3. nonlinearities within perturbation theory, construed as the gravity's contribution to its own source, and 4. the Noether Theorems, linking symmetries and conserved quantities. As it stands, each argument is found to be either contentious, or incomplete in that it presupposes substantive assumptions which the standard exposition glosses over. I finally investigate the standard interpretation of binary systems, according to which orbital decay is explained by the system's energy being dissipated via GW energy-momentum transport. I contend that for the textbook treatment of binary systems an alternative interpretation, drawing only on the general-relativistic equations of motions and the Einstein Equations, is available. It's argued to be even preferable to the standard interpretation. Thereby an inference to the best explanation for GW energy-momentum is blocked. I conclude that a defence of the claim that GWs carry energy can't rest on the standard arguments.

Key words: General Relativity, Gravitational Waves, Sticky Bead, binary systems, Problem of Motion

II.1. Introduction

Do gravitational waves (GWs) carry energy-momentum? Many will deem this old hat. A contemporary monograph on GW astrophysics representatively proclaims: "Since gravitational waves produce a real physical effect […] – it is clear that the wave must be carrying energy" (Anderson & Creighton, 2011, p. 66). As a paradigmatic, "real physical effect" textbook orthodoxy adduces the orbital spin-up of double pulsars, discovered by Hulse and Taylor in 1974. On the standard interpretation, the double pulsars are evidence for the energy which the GWs have radiated away. As a result, the system loses energy, and the pulsars' orbits (equivalently: their orbital periods) decrease. This prediction has been corroborated with stunning accuracy (Will, 2014).

Since the recent (repeated) direct detection of GWs, the *existence* of GWs is beyond reasonable dispute (cf. Castelvecchi & Witze, 2016). But perhaps the received *interpretation* that GWs transport energy and thereby deplete the binaries' orbital energy deserves further reflections. Three thoughts that could seed some doubt might spring to mind.

One thought is that the pulsars (modelled as dust particles) are in free-fall. Hence they move inertially. Shouldn't their kinematic state therefore remain unaltered? In particular, shouldn't the binaries preserve their energy (Petkov, 2012, Appendix C)?

More generally, Norton (2012, sect. 3.9) has remarked: if in GR the gravitational force is "geometrised away" shouldn't this compromise the notion of gravitational energy, as well?

Closely related is a third thought, touching on energy-momentum conservation. As we know from Analytic Mechanics, conservation of energy and momentum is tied up with the symmetries of space and time (see, e.g., Landau & Lifshitz, 1976, §6). But isn't space-time, according to GR, warped, generically lacking any symmetries? Shouldn't this affect the validity of conservation laws?

In this chapter, I critically examine the four standard arguments found in the astrophysics literature for ascribing GWs energy-momentum. I deny that these arguments succeed – at least, as they stand. The first two adduce phenomenological effects that GWs produce: the "argument from kinetic energy" concerns the change in kinetic energy of a ring of free particles hit by GWs (**§2.1**). The second effect, the "Sticky Bead Argument", consists in the heating-up of a detector whose constituents are held together by internal forces (**§2.2**). Of a more systematic nature are two further arguments: the natural energy-momentum of a GW, obtained from a decomposition of the metric into a background and perturbations (**§2.3**), and an application of Noether's Theorem (**§2.4**), respectively. If the standard arguments for ascribing GWs energy-momentum fail, how then to account for their effects? **§3** turns to the paradigmatic treatment of binary systems (**§3.1-2**). In (**§3.3**) I sketch how their problematic standard account in terms of the energy that GWs are supposed to transport can be replaced by an attractive alternative. It solely appeals to the general-relativistic equations of motion and the Einstein Equations. Thereby, an inference to the best explanation for GW energy-momentum is blocked (**§3.4**). I outline two promising lines of further inquiry in **§4**.

Clearly, if standard textbook arguments are unsatisfactory, greater attention –especially for non-experts and students – should be drawn to their lacunae (and how they might be filled). But the ramifications of my analysis are wider-reaching.

One concerns the role of the standard arguments in *motivating* other approaches. My discussion of the standard arguments remains restricted to only a modicum of mathematical rigour. (Criticising the standard arguments for lack of rigour *per se* would miss the mark.) Nonetheless, they play an essential role in motivating mathematically more sophisticated approaches to GW energy, e.g. ADM energy, Komar mass or the Bondi-News-Function (for details, see e.g. Jaramillo & Gourgoulhon, 2010). In his state-of-the art monograph, Maggiore (2008, p. 26 my emphasis), for instance, explicitly avers that the first two standard arguments are *conclusive*: "The fact that GWs indeed carry energy and momentum is *already clear* from the discussion of the interaction of GWs with test masses presented above."

In addition to their (as I'll argue: dubious) motivational role, the standard arguments also play a suspect role as reasons for thinking that those more sophisticated approaches possess *physical significance*. The belief that they do largely rests on the standard arguments. Serving as exemplars in the sense of Kuhn (2012, esp. postscript), they shape our hunches for what counts as physical, and not merely formal. But hunches aren't arguments, of course.

The alternative, purely dynamical interpretation of the binary problem (laid out in §3) further aggravates the issue. On several explanatory virtues, the dynamical interpretation trumps the received one. Future work will have to gauge whether the former's superiority carries over to explaining other GW phenomena (e.g. black hole merger scenarios). Even so, the existence of an alternative account subverts the naïve idea that explanatory utility of certain formal quantities justifies realist commitment towards them (see e.g. Norton, 2018). Consider, for instance quasi-local definitions of gravitational energy. They are useful in computing tidal heating (see, e.g. Szabados, 2012, §13.1). But unless the standard arguments already persuaded one of the existence of GW energy, utility by itself isn't enough to vindicate their physical significance.

A proviso is in order. It concerns my prerequisite ontological and ideological commitments. My analysis presupposes GR's standard "minimally geometric" interpretation: gravitational effects are re-conceptualised as manifestations of non-Minkowskian inertial structure (see e.g. Norton, 2012, pp. 19; Nerlich, 1994, Ch. 7; Nerlich, 2013, esp. Ch. 8, 9). This minimally geometric interpretation affords us neutrality on the "relative explanatory priority of geometry (the orthodox view [the 'geometric interpretation', as ordinarily understood, P.D.]) or the dynamical laws (the dynamical/constructivist view)" (Pitts, 2017, p. 13). By the same token, I remain non-partisan as to whether inertial structure is ontologically dependent on (or even reducible to) matter. (In other words: I steer clear of the debate over the dynamical/geometrical approach to spacetime, see, for instance, Brown & Read, 2018.) For my purposes, the platitude suffices that GR possesses inertial structure, i.e. distinguished states of "natural" motion, characterised in broadly functionalist terms (cf. Knox, 2017b). Thereby the intricacies of more specific commitments can be by-passed. At the same time, the minimally geometric interpretation captures GR's distinctive intertwinement of gravity, inertia and chronogeometricity. Gravitational energy and the alleged energy of GWs in particular sit at the heart of this intertwinement. It's my hope that the line of thoughts developed below will enhance a better understanding of it. This in turn may bear upon a principled assessment of other interpretative questions in GR (cf. Lehmkuhl, 2008; Rey, 2013).

This chapter thus aims at conceptual analysis: given minimal ideological and ontological commitments about GR as a fundamental theory, do the textbook arguments for (local) GW energy hold water? In particular, I wish to sensitise the reader to the question: to what extent do arguments for GWs that employ approximation schemes (e.g. the PPN formalism) go beyond those minimal commitments?

II.2 Four Routes to Gravitational Wave Energy

In this section, I revisit and re-evaluate the four canonical arguments found in the astrophysics textbook literature for ascribing GWs energy-momentum. They rest on the following respective ideas:

(1) GWs incident on free particles, initially at rest in a lab frame, set them into motion.

(2) GWs can induce heat in a detector.

(3) Within perturbation theory, higher-order contributions can be naturally construed as the GWs' energy-momentum.

(4) The framework of the Noether Theorems, applied to GW theory, leads to energy-momentum in a way fully analogous to other field theories.

None of the arguments, I maintain, is compelling. They either rest on conceptual distinctions not available within GR, with its standard ontological and (geometric) ideological commitments mentioned above. This the case with (1) and (3). Or – as is the case with (2) and (4) – they implicitly hinge on non-trivial assumptions that require (at least) substantive additional arguments.

II.2.1. Kinetic energy of test masses

The default argument for the energy of a GW turns on the effects of GWs upon test particles, otherwise at rest. Their increase in kinetic energy is supposed to have been extracted from the GW.

The argument employs a perturbative treatment of GR, so-called linear GW theory (for details, see e.g. Misner, Thorne & Wheeler, 1973, Ch.35; Hobson, Efstathiou & Lasenby, 2006, Ch. 17). (The perturbative approach will be studied in full generality in §2.3.) Within linearised theory, one assumes that the gravitational field is weak. The spacetime metric, g_{ab}, then deviates only slightly and slowly from a flat Minkowski background η_{ab}:[7]

$$g_{ab} = \eta_{ab} + h_{ab}, \text{where } |h_{ab}|, |\partial h_{ab}|, |\partial^2 h_{ab}|, \ldots \ll 1.$$

[7] For GWs incident on our GW detectors $h_{\mu\nu}$ is typically of order 10^{-21}. For comparison, the absolute values of gravitational fields in our solar system are still quite small, typically: $|h_{ij}| \lesssim 10^{-6}$ (for details, see Misner, Thorne & Wheeler, 1973, Ch. 39).

All terms beyond linear order in the metric perturbations or their derivatives are discarded. In this perturbative order, one can treat h_{ab} as a symmetric tensor field under global Lorentz transformations. (Henceforth in this chapter (II), Greek indices will denote approximately Lorentz tensors. Indices thus are also raised/lowered with respect to the Minkowski metric.) Linearised theory thus is effectively a special-relativistic theory of gravity for weak fields (cf. Ohanian & Ruffini, 2013, Ch. 3,4). (Concomitantly, one introduces Cartesian coordinates as inertial coordinate systems. They privilege global Lorentz transformations: only under them do objects preserve their invariance.)

Expanding the general-relativistic tensors in powers of the perturbation $h_{\mu\nu}$ yields (to leading order) the corresponding quantities in linearised theory (denoted by the scripted symbols). As an example, consider the linearised Einstein tensor:

$$\mathcal{G}_{\mu\nu} = \partial^2_{\mu,\nu} h + \Box h_{\mu\nu} - \partial^2_{\lambda,\nu} h^{\lambda}_{\mu} - \left(\Box h - \partial^2_{\kappa,\lambda} h^{\kappa\lambda}\right)\eta_{\mu\nu},$$

with $h \coloneqq \eta^{\mu\nu} h_{\mu\nu}$ and the flat spacetime d'Alembertian $\Box \coloneqq \eta^{\mu\nu}\partial^2_{\mu,\nu}$. For consistency, the energy-momentum tensor $T_{\mu\nu}$ on the r.h.s. of the Einstein Equations must likewise be of first order in the perturbations ($T_{\mu\nu} \approx \mathcal{T}_{\mu\nu}$).

Harnessing the gauge freedom for the gravitational field, the linearised Einstein Equations, $\mathcal{G}_{\mu\nu} = -\frac{16\pi G}{c^4}\mathcal{T}_{\mu\nu}$, simplify for a particular gauge (the so-called "TT-gauge") to an inhomogeneous wave equation. For the purposes of GWs, we may restrict ourselves to the vacuum case ($\mathcal{T}_{\mu\nu} = 0$) with plane wave packets as solutions. They have the form $h_{\mu\nu} = \int d^3\vec{k} A_{\mu\nu}(\vec{k}) e^{ik_\lambda x^\lambda}$, with a generic, wave-vector dependent function $A_{\mu\nu}$, the so-called polarisation tensor.

One might baulk at imposing the above TT-gauge condition: how to ensure that these waves aren't gauge artefacts? Within the so-called "cosmological perturbation formalism" it can be shown that imposing the TT-gauge condition doesn't curtail the general validity of the argument. Only the transverse, traceless degrees of freedom of the metric genuinely (i.e. not as an illusory artefact of a coordinate choice) obey a wave-like equation; only they can be said to propagate in a physical sense. Hence, one can identify them as radiative (for details see, e.g., Flanagan & Hughes, 2005). (The other components satisfy an equation of the Poisson type. They represent static degrees of freedom.)

Like an electromagnetic wave, a GW possesses two linear (or two circular) polarisations. Their respective designations, ⊕ and ⊗, indicate the axes along which a ring of a test particles is distorted. The effect that a purely ⊕-polarised GW travelling along the z-axis, $h_{ab}^{\oplus} = \cos k(ct - z)\, e_{ab}^{\oplus}$ (with the polarisation tensor $e_{ab}^{\oplus} = diag(0,1,-1,0)$) produces in a transverse circle of particles is illustrated in Figure 1.

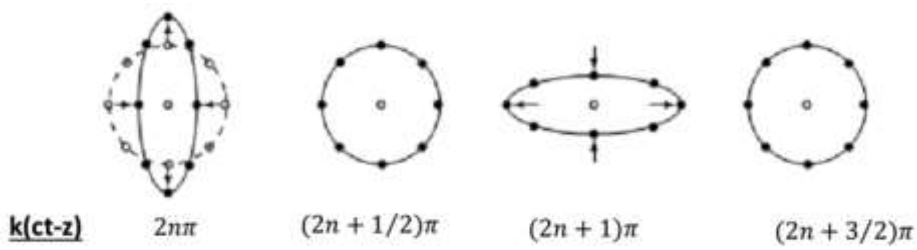


Fig. 1: Deformations of a ring of test particles in the x-y plane. The initial configuration is shown by the open dots. $n \in \mathbb{Z}$

For laboratory practices, it's expedient to adopt the so-called proper detector frame (PDF), a Fermi coordinate system along a geodesic. (For details about the PDF and TT frame, see Maggiore, 2008, Ch. 1.3.3.)

In the coordinate system adapted to the PDF, one fixes the origin on a free-fall trajectory, and then uses rigid rulers to delineate coordinates. The geometry measured in these coordinates is Euclidean. (Such rulers are rigid only in linear order on suitable (material-dependent) length scales small relative to the GW wavelength, op.cit., fn 11.) Imagine a laboratory on a (drag-free) satellite in free-fall in the Earth's gravitational field. Then, restricting ourselves to a sufficiently small region (small, compared to the curvature radius $\mathcal{R} = |R_{abcd}|^{-1/2}$), we can choose coordinates x^{λ} (viz. Fermi normal coordinates) such that the metric is flat even in the presence of a GW,

$$ds^2 \approx \eta_{\mu\nu} dx^{\mu} dx^{\nu} + \mathcal{O}\left(\frac{|\boldsymbol{x}|^2}{\mathcal{R}^2}\right).$$ [8]

Ground-based detectors with characteristic sizes much smaller than the GW's wave length can be effectively described in the PDF. (Its applicability ceases for space-based detectors with their usually large spatial extension.)

[8] By contrast, for the above purely ⊕-polarised plane GW, the line-element in the TT-gauge takes the form:

$$ds^2 \approx c^2 dt^2 - (1 - \mathrm{h}^{\oplus} \cos k(ct - z))dx^2 - (1 + \mathrm{h}^{\oplus} \cos k(ct - z))dy^2 - dz^2.$$

How does an incident GW of amplitude $h_\oplus$ affect the above ring of test particles (each assumed to be of mass m), when one adopts the PDF? The GW deforms the ring, stretching and squeezing it in the x- and y-direction, respectively: $\delta x(t) = \frac{h_\oplus}{2}\sin\omega t$ and $\delta y(t) = -\frac{h_\oplus}{2}\sin\omega t$. (Fig.1 depicts the deformations by the black arrows.) The particles start moving, thereby changing their kinetic energy:

$$E_{kin} = \frac{m}{2}(\delta\dot{x}^2 + \delta\dot{y}^2) = \frac{m}{4}h_\oplus^2\omega^2\cos^2\omega t.$$

Whence this energy gain – the argument from kinetic energy runs – if not from the GW?

One may object to this argument on two grounds. Firstly, it utilises coordinates that *aren't* adapted to the inertial frames. Secondly, one may criticise the ambiguity of "kinetic energy" in a GR context.

At the core of the argument from kinetic energy lies its reliance on the PDF. This reference frame isn't physically privileged in linearised GR, however: its adapted Fermi coordinates are inertial coordinates only along *one particular* free-fall trajectory (e.g. one selected particle). For *all other* free-fall particles the PDF's coordinates aren't adapted: they aren't inertial coordinates with respect to the *effective* metric, $\eta_{\mu\nu} + h_{\mu\nu}$. (It's instructive to re-phrase the problem from an operationalist perspective. With the PDF's coordinates being delineated by rigid rulers, according to a theorem by Helmholtz, one can't physically realise a PDF (not even in first order!) in spaces of variable curvature (see Mittelstaedt, 1981, Ch. II, §3), including those with GWs – unless additional forces are posited ad-hoc to counteract the deformations that the curvature inflicts on extended bodies.)

The so-called TT-frame is no less natural.[9] It's realised by test particles in free-fall. The labels of the coordinates adapted to the TT-frame *comove* with the test particles, i.e. along geodesics. (That these coordinates are the ones satisfying the TT-gauge condition introduced earlier, follows directly from the geodesic equation.) Hence, for our ring of test particles the TT-frame is distinguished *globally*: its coordinates are adapted to *all* particles in free-fall frames.

[9] Kennefick (2007, p. 131) likens the TT-frame to the natural way oceanographers would introduce coordinates (in the form of buoys) for orientation on sea.

In the TT frame, no particle in the above scenario changes its coordinate position, $\frac{dx^a}{d\tau} = 0$ (see, e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 18.4). The GW doesn't affect the particles' kinetic energy. This contradicts the previous conclusion from the argument from kinetic energy. How to resolve this paradox?

One might object that the TT-frame isn't a global inertial frame: not *all* the laws of physics take a particularly simple form in it. (Not all of the Christoffel symbols vanish.) But it's unclear –not least, in light of GR's general covariance – why this should prevent us from choosing of the associated coordinates: shouldn't physical quantities be independent of the *conventional* choice of coordinates?

The confusion originates in the notion of kinetic energy: it's frame-dependent. But the above argument imported the *Newtonian* notion of kinetic energy. Conceptually, this isn't licit. To fathom the real force of the argument from kinetic energy, we must first identify the *general-relativistic* counterpart of kinetic energy. Then, we can re-run the argument.

In Classical Mechanics, kinetic energy plays two distinct roles. On the one hand it's the residual part of the total energy after subtracting the energy contributions from all interactions. On the other hand, it's the (numerical value of) the Lagrangian whose variation yields the equation of motion of free particles.[10] Both roles are only contingently related. In the transition to general-relativistic mechanics, they are no longer played by the same object.

Which quantity takes over the role of "non-interactional energy"? Consider the energy $E[\xi] = g_{ab}p^a\xi^b$ of a particle with mass *m* and the 4-momentum p^a, relative to an observer with the 4-velocity ξ^a.[11] (For simplicity, let's ignore all non-gravitational interactions, as well as contributions from static gravitational fields.) Subtracting from this energy the particle's rest energy yields the (generally covariant) relativistic kinetic energy:

$$E_{\text{kin}} := p_a\xi^a - mc^2 = mg_{ab}u^a\xi^b - mc^2$$

To evaluate this further, consider the norm of the particle's 4-velocity,

$$c^2 = g_{ab}u^a u^b = c^2 g_{00}\left(\frac{dt}{d\tau}\right)^2 + 2cg_{0i}v^i\frac{dt}{d\tau} + c^2 g_{ij}v^i v^j\left(\frac{dt}{d\tau}\right)^2.$$

[10] This is how e.g. Landau & Lifshitz (1976, §4) define kinetic energy.

[11] This mimics the derivation of the mass-energy equivalence in SR, e.g. in Malament (2012, Ch.2.4).

Here, τ denotes its affine parameter and $v^j = \frac{dx^j}{dt}$ the coordinate velocity.

Recalling that for a GW in TT-gauge passing through the particle, $g_{0i} = 0$, one obtains:

$$u^0 = c\frac{dt}{d\tau} = \frac{c}{\sqrt{g_{00} + g_{ij}v^i v^j}}.$$

For an inertial observer at rest, $\xi^i = 0$, the norm of the observer's 4-velocity entails that $\xi^0 = \frac{c}{\sqrt{g_{00}}}$. Plugging the expressions for u^0 and ξ into the above general-relativistic kinetic energy yields:[12]

$$E_{kin} = mc^2\left(\sqrt{\frac{g_{00}}{g_{00} + g_{ij}v^i v^j}} - 1\right).$$

Adopt now the TT-frame. In it, the initial particle positions don't change, $v^j = 0$. Consequently, the particle *doesn't* gain kinetic energy! The argument from kinetic energy is short-circuited.

What about the GR counterpart of the second notion of kinetic energy? The GR Lagrangian whose variation entails the equation of motion for a free massive particle takes the same form as in SR:

$$\mathcal{L}_0 = \sqrt{\left|g_{ab}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau}\right|}.$$

When integrated along a worldline between two points, $\mathcal{L}_0$ yields the particle's proper distance. While the presence of a GW surely changes the latter, $\mathcal{L}_0$'s numerical value remains unchanged: viz. *c*. ($\mathcal{L}_0$ transforms like a scalar. To get its numerical value, we can hence evaluate it in any arbitrary coordinate systems, including those adapted to the TT frame.) Again, the argument from kinetic energy is short-circuited.

In summary, the argument from kinetic energy has two key flaws. Firstly, it uses a naïve *Newtonian* notion of kinetic energy. Secondly, it bestows on a non-adapted coordinate system

[12] It has the limit of the special-relativistic expression for kinetic energy $mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$.

a privileged status that prima facie doesn't seem justified. We remedied the argument by identifying the two quantities that take over the respective roles of kinetic energy. Neither increased during the passage of a GW.

An implicit premise of the argument from kinetic energy is energy conservation: *if* a free system's kinetic energy *were* changed, energy conservation *would* imply that the GW had imparted energy to the system. Is this assumption valid? To this we now turn by discussing a related phenomenological argument, the Sticky Beads Argument.

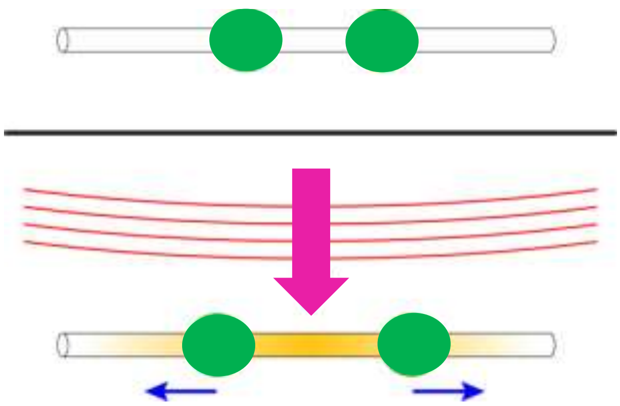
II.2.2 Bondi-Feynman's Sticky Bead Argument

In the 1950s, the emission of GWs was still fiercely debated (for the history of this debate, see Kennefick, 2007, Ch. 5-7). As a historical matter of fact, the controversy in the physics community was settled by a simple, qualitative thought experiment, proposed independently by Bondi and Feynman (cf. Feynman, 2002, Foreword). It appears to demonstrate compellingly that GWs must carry energy and *hence* must be real.[13]

The idea is that a GW can heat up matter. The thermal energy, the Sticky Bead Argument goes, is extracted from the GW energy.

Fig. 2:

Upper part: sticky beads at rest
Lower part: The incident GW sets the beads in motion along the stick, which causes friction. The stick heats up (yellow).



More precisely, consider beads on a stick, serving as a detector. The two beads can "[slide] freely (but with a small amount of friction) on a rigid rod. As the wave passes over the rod,

[13] It's worth pointing out that some authors – explicitly e.g. Bunge (2017) – seem to regard energy transport of GWs also as a *necessary* criterion for their existence.
The view I advocate is that, while I don't deny that energy transport *would* constitute a sufficient criterion for the reality of GWs, I don't regard it as a necessary one. GWs in my opinion *are* real, and they manifest themselves in real phenomena, including changes in matter energy-momentum. They *don't* involve, however, any energy-momentum *exchange* between the GW and matter. GWs exist, but they needn't carry energy-momentum.

atomic forces hold the length of the rod fixed, but the proper distance between the two beads oscillates. Thus, the beads rub against the rod, dissipating heat" (Feynman, Moringo & Wagner, 2002, p. xxv-xvi). According to Feynman, the subsequent heating up shows that the GW can do work: for conservation of energy to hold, whence should the gain in thermal energy stem, if not from the GW? According to the Sticky Bead Argument, GWs must therefore be ascribed energy-momentum.

To discern more clearly which assumptions underlying this reasoning are potentially problematic, let's render the argument more quantitative. (I'll follow the discussion in Anderson & Creighton (2012, pp. 65).) Replace the beads on a stick with a simple damped spring. Now consider two masses *m₁* and *m₂*, placed on the *x*-axis, and connected by a spring of spring constant *k*. When the masses are separated by length *L*, the spring is at equilibrium. Let *x* measure the displacement of the masses with respect to this equilibrium. If a purely $\oplus$-polarised GW hits the system, the induced oscillations obey $\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 = -\frac{1}{2}h_{\oplus}L\omega^2\cos\omega t$ with the characteristic frequency of the oscillator $\omega_0 \coloneqq \sqrt{\frac{k}{\mu}}$, the reduced mass of the system $\mu \coloneqq \frac{m_1 m_2}{m_1+m_2}$ and the damping parameter $\beta \coloneqq \frac{b}{2\mu}$, where the dissipative force is assumed to be $F_{\text{diss}} = -b\frac{dx}{dt}$. The work done by the GW on the oscillator, averaged over a cycle of oscillation, can thus be determined to give $\langle W_{GW}\rangle = -\langle E_{\text{kin}} + E_{\text{pot}}\rangle = -\langle\frac{\mu}{2}\left(\frac{dx}{dt}\right)^2 + \frac{k}{2}x^2\rangle = \beta\mu x_{\max}^2\omega^2$ with the resonant amplitude $x_{\max} = \frac{1}{2}\frac{h_{\oplus}L\omega^2}{\sqrt{(\omega_0^2-\omega)^2+4\omega^2\beta^2}}$. This dissipated energy manifests itself as thermal energy. The changes in total energy of the system, $E_{\text{kin}} + E_{\text{pot}}$, are counterbalanced by changes in the energy of the GW. One can thus determine these changes in the GW's energy from the dissipation.

The Sticky Bead Argument looks compelling – provided one subscribes to two premises:

(1) General-relativistic correction terms on the internal structure of the stick (in particular its binding energy) are negligible (Cooperstock & Tieu, 2012, pp. 85). In our qualitative version of the argument this corresponds to assuming that the spring constant (which depends on the internal structure of the spring) remains the *same* before and after the GW detection.

(2) Energy conservation holds: an increase of energy is always *counterbalanced* by a decrease of energy elsewhere. This principle was expressly invoked in both versions of the Sticky Bead Argument.

No argument, however, was given for either premise. With reason, one may impugn both. Cooperstock and Tieu (ibid.), for instance, have – *pace* Feynman (cited in: Kennefick, 2007, p. 136) – attacked (1). They claim that no heat transfer occurs, when one properly models the stick: "what has been overlooked is that the bar itself has been presumed to be unaffected by the gravity waves".

To-date, a satisfactory response to Cooperstock's efforts to rebut the Sticky Bead Argument is still pending (Kennefick, 2007, p. 254).[14] Even so, I won't discuss the physically adequate modelling involved the Sticky Bead Argument. Rather, my focus will be on (2): in GR, energy conservation can scarcely be just *presumed*. On the contrary: The violation of energy-momentum conservation for non-flat spacetimes is widely countenanced in the GR literature (e.g. Eddington, 1923, pp. 135; Schrödinger, 1950, pp. 72; Weinberg, 1972, p. 166; Misner, Thorne & Wheeler, 1974, §19.4; Padmanabhan, 2010, p. 213; Hoefer, 2000; Lam, 2011).

More precisely, only highly symmetric spacetimes allow for satisfactory statements of energy conservation. If and only if a spacetime possesses a time-like so-called Killing vector ξ (satisfying $\nabla_{(a}\xi_{b)} = 0$), is the energy-momentum current $j^a[\xi] \coloneqq T^a_b \xi^b$ in the direction of ξ locally (differentially) conserved, i.e. possesses no sinks or sources:

$$\nabla_a j^a[\xi] = 0.$$

Only when a spacetime has Killing vectors, can the local conservation law be converted into a global (integral) one. The energy-momentum enclosed within a space-like hyperplane Σ_τ is then independent of the foliation of the spacetime, and doesn't vary with time:

$$\frac{d}{d\tau}\int_{\Sigma_\tau} d^3x\,\sqrt{|g|}\,j^a[\xi] = 0.$$

[14] One surely ought not to underestimate the subtleties of the interplay between gravity and electromagnetism Think, for instance, of the delicate question whether a point charge in free-fall radiates (cf. , for instance, Lyle, 2008).

Nowhere in the Feynman-Bondi Sticky Bead Experiment has the existence of a time-like Killing vector been mentioned explicitly. Nor could it have been: a time-like Killing vector essentially means that the metric is time-independent. Such a spacetime is static– contrary to the idea of a GW as a curvature effect *propagating* through spacetime.

It's possible to relax the requirement of Killing vectors. One can still formulate useful conservation laws, if the spacetime has *asymptotic* Killing vectors – certain suitably defined symmetries at infinity (for details, see e.g. Geroch, 2013, Ch. 36-38; Wald, 1984, Ch. 11; Jaramillo & Gourgoulhon, 2010). A crucial assumption here is asymptotic flatness: The metric must, roughly speaking, fall off sufficiently fast, approximating the Minkowski metric at infinity. This imposes *substantive* restrictions on the space of admissible spacetimes. Their justification isn't unproblematic. I'll revert to this in §2.4. Suffice it here to mention that, while a successful idealisation in many astrophysical contexts, in others asymptotic flatness can no longer be assumed. For instance, no perfect fluid model of a rotating star that can be matched in the exterior to the (asymptotically flat) Kerr solution is known. In fact, a counterexample of a rotating perfect fluid exists, the so-called Wahlquist fluid. More importantly, in no realistic scenarios does asymptotic flatness hold any longer: our ΛCDM-universe isn't asymptotically flat – not even approximately. Instead, it's asymptotically deSitter. Asymptotic flatness should thus be viewed as an *idealisation* in the sense of Norton (2011): within a certain regime, it adequately models some aspects of subsystems of our universe by dint of a *distinct* surrogate system which approximately *mimics* distinctive features of the target system. But with that, the predictive and explanatory success of asymptotically flat idealisations no longer warrants an unqualified realism about all explanantia involved – in particular, about GW energy. Should one wish to uphold a realism about it, one would have to resort to the "selective move" (cf. Vickers, 2017): asymptotic flatness would have to be shown to be a "working posit" of (i.e. essential for) relativistic astrophysics. But this is questionable. What matters is to obtain an approximate solution of the Einstein Equations (and the corresponding general-relativistic equations of motions) for a given scenario. Despite its occasional computational convenience, there is no reason why asymptotic flatness should be viewed as an indispensable boundary condition (especially not, given that our universe just isn't asymptotically flat).

Therefore, *even if* (1) were warranted and we did register an increase in thermal energy of a Sticky Bead detector, we wouldn't be licenced to infer a *transfer* of energy from the GW, so as to restore energy balance. Rather, it would seem more natural to accept an alternative stance: energy conservation simply ceases to hold in GR. The detector would just heat up - without there being a *causal* story about it that would allow us to track the lost energy. Energy conservation is just *violated* (quantifiably!), when a GW hits a detector. (More on this in §3.2.)

In summary, the cogency of the Sticky Bead Argument derives from premises that, albeit uncontroversial in pre-GR physics, are contentious - and plausibly false in GR.

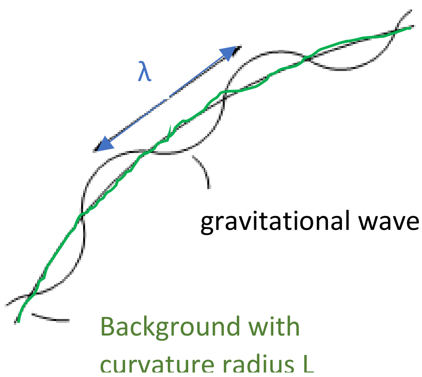
II.2.3. Perturbative approach

We already encountered linearised GW theory in both the argument from kinetic energy (§2.1) and the Sticky Bead Argument (§2.2). There, the metric was expanded around a flat background. Perturbation theory generalises this idea by decomposing it into a slowly-varying background and a fast-varying perturbation. The latter then is identified as the GW – a ripple of spacetime. (For details of what follows below, see Maggiore, 2008, Ch. 1.4; Padmanabhan, 2010 , Ch. 9.5.) An object then emerges that appears a natural candidate for representing the GW's energy-momentum.

For simplicity, we restrict ourselves to vacuum solutions of the Einstein Equations, $G_{ab} = 0$. Let there exist a suitable length or time scale of the variation. This enables us to decompose the metric into a background and small fluctuation components.

Fig. 3:

Characteristic length scales λ of the GW and of the background curvature L, respectively



Applying the standard scheme for perturbation theory to the metric (with the formal book-keeping parameter ϵ),

$$g_{ab} = g_{ab}^{(0)} + \epsilon g_{ab}^{(1)} + \epsilon^2 g_{ab}^{(2)},$$

the Einstein tensor can be expanded up to $\mathcal{O}(\epsilon^2)$ as

$$G_{ab} = G^{(0)}_{ab}\left[g^{(0)}_{ab}\right] + \epsilon G^{(1)}_{ab}\left[g^{(0)}_{ab}, g^{(1)}_{ab}\right] + \epsilon^2 \left(G^{(1)}_{ab}\left[g^{(0)}_{ab}, g^{(2)}_{ab}\right] + G^{(2)}_{ab}\left[g^{(0)}_{ab}, g^{(1)}_{ab}\right]\right).$$

Here, the superscript "(0)" denotes the unperturbed (0th-order) quantities and, correspondingly, "(1)" the 1st-order perturbations, etc. The dependence of the 1st-order Einstein tensor, $G^{(1)}_{ab}\left[g^{(0)}_{ab}, g^{(1)}_{ab}\right]$, on the arguments in brackets signifies that it's built from 0th and 1st-order terms of the metric. Terms of each order are assumed to vanish separately.

In the expansion of the Einstein tensor, the first two terms describe the unperturbed background geometry, and the evolution of the perturbations on the background (i.e. the GWs) respectively. Of principal interest for us is the third term: It describes how the 2nd order perturbations are related to the background and 1st-order perturbations. Recast as

$$G^{(1)}_{ab}\left[g^{(0)}_{ab}; g^{(2)}_{ab}\right] = \frac{8\pi G}{c^4} t^{(\mathrm{eff})}_{ab},$$

with the effective GW energy-momentum tensor $t^{(\mathrm{eff})}_{ab} = -\frac{c^4}{8\pi G} G^{(2)}_{ab}\left[g^{(0)}_{ab}; g^{(1)}_{ab}\right]$ on the r.h.s., it lends itself to an intuitive interpretation: the 2nd-order perturbations of the metric are *sourced* by the effective GW energy-momentum. This reflects, one is tempted to think, the back-reaction of the gravitational field upon itself.[15] That is: on this view, gravitational energy *qua* energy contributes to the generation of its own field; "gravity itself gravitates."

A blemish taints $t^{(\mathrm{eff})}_{ab}$, though: the effective gravitational energy-stress pseudo-tensor isn't invariant under the local gauge transformations of the type $h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\xi_{\nu)}$, for an arbitrary ξ^a. (For simplicity, assume the spacetime background to be flat.) This lack of gauge-invariance can be cured (see e.g. Misner, Thorne & Wheeler, 1973, §35.15) by averaging over a 4-volume $\Delta\mathcal{V}$ of several wave lengths of the GW (but still smaller than the length scale of variation of the background):

$$T^{(\mathrm{GW})}_{ab} := \langle t^{(\mathrm{eff})}_{ab} \rangle = \frac{1}{|\Delta\mathcal{V}|}\int_{\Delta\mathcal{V}} d^4x \sqrt{|g^{(0)}|}\, t^{(\mathrm{eff})}_{ab}.$$

(Smoothing out $t^{(\mathrm{eff})}_{ab}$ doesn't affect the actual physics: perturbations can only be defined with respect to the *typical* length/time scales of the background, anyway.) This averaged GW

[15] For the sake of the argument, I grant that the gravitational field should indeed be identified with the metric – thereby glossing over a prolonged debate (see Lehmkuhl, 2008).

energy-momentum then is gauge-invariant. (The changes of $t_{ab}^{(\text{eff})}$ resulting from gauge transformations take the form of total divergences. They are eliminated by integration.)

$T_{ab}^{(\text{GW})}$ has several properties that at first blush invite its interpretation as a GW's energy-momentum. 1. It transforms covariantly w.r.t. tensor transformations of the *background* metric $g^{(0)}$. (Indices therefore are also raised/lowered with respect to $g^{(0)}$.) 2. By construction, it's symmetric (a requirement for defining angular momentum). 3. It obeys a generally covariant conservation law, $\nabla^{(0)b}\left(T_{ab} + T_{ab}^{(\text{GW})}\right) = 0$, where the covariant derivative is defined with respect to the *background* metric, $\nabla^{(0)b} g_{ab}^{(0)} = 0$. (N.B.: The covariant divergence of the matter energy-momentum tensor vanishes separately with respect to the connection compatible with the *full* spacetime metric simpliciter, i.e. the background *plus* the perturbations: $\nabla^b T_{ab} = 0$.) 4. Like other energy-momentum tensors from classical field theories, it's quadratic in the dynamical field variables (here: the perturbations $g_{ab}^{(1)}$).

It appears to originate in the non-linearity of the Einstein Equations – in accordance with the slogan that "gravity gravitates": all forms of energy (including gravitational energy itself) act as a source for the gravitational field.

An equivalent re-formulation of the perturbative approach in terms of a variational principle ("Isaacson's approach", Schutz & Ricci, 2010, sect. 4.2) is instructive. The idea is to expand the *action* $S[g_{ab} + h_{ab}] = \int d^4x \sqrt{|g[g_{ab} + h_{ab}]|} R[g_{ab} + h_{ab}]$ with respect to the perturbations h_{ab} around the background g_{ab}, such that:

$$\begin{aligned} S[g_{ab} + h_{ab}] = S[g_{ab}] &+ \int dx^4 h_{ab} \frac{\delta\left(\sqrt{|g|}R\right)}{\delta g_{ab}} \\ &+ \frac{1}{2}\int dx^4 \left(\frac{\partial^2\left(\sqrt{|g|}R\right)}{\partial g_{ab}\partial g_{cd}} h_{ab} h_{cd} + 2\frac{\partial^2\left(\sqrt{|g|}R\right)}{\partial g_{ab}\partial(\partial_e g_{cd})} h_{ab}\partial_e h_{cd} \right. \\ &\left. + \frac{\partial^2\left(\sqrt{|g|}R\right)}{\partial(\partial_e g_{cd})\partial\left(\partial_f g_{cd}\right)} \partial_e h_{cd}\partial_f h_{cd} + 2\frac{\partial^2\left(\sqrt{|g|}R\right)}{\partial g_{ab}\partial\left(\partial_{e,f} g_{cd}\right)} h_{ab}\partial_{e,f} h_{cd} \right) + \mathcal{O}(h^3). \end{aligned}$$

The term in brackets in the last integral can be regarded as (proportional to) the GW-Lagrangian, $32\pi L_{(GW)}$. It's a function of the background metric and its perturbation, $L_{(GW)} = L_{(GW)}(g_{ab}, h_{ab})$.

Via this GW-Lagrangian, one can define an effective energy-stress tensor associated with the GWs as a variational derivative with respect to the background metric:

$$t^{ab}_{(GW)} = \frac{2}{\sqrt{|g|}} \frac{\delta}{\delta g_{ab}} \left(\sqrt{|g|} L_{(GW)}\right).$$

Averaging as before yields the same effective energy-momentum as the previous expression:

$$T^{ab}_{(GW)} = \langle t^{ab}_{(GW)} \rangle.$$

Does this supply a convincing proposal for GW energy? Two reasons discourage a positive answer. The first turns on the fact that the perturbative approach relies on stipulations unwarranted within GR's conceptual framework. The second consists in the limited utility of perturbatively defined GW energy.

The perturbative approach explicitly presupposes a background-perturbation split, both in the construction of the energy-momentum from perturbative orders, and in the averaging procedure to remedy gauge-dependence. Such a split is occasionally justified from a pragmatic point of view, say for primordial GWs on an FLRW background. In such cases, one deals with discriminable scales, λ and L, over which the background geometry and the perturbation vary, $\lambda \ll L$.

As Padmanabhan (2010, p. 420; cf. also Padmanabhan, 2004) insists, however, a straightforward comparison of the orders of magnitude discloses that "one cannot introduce the concept of a gravitational wave of arbitrarily large amplitude but varying at a length scale that is sufficiently small compared with the background scale of variation and develop a systematic perturbation theory". For instance, at early times during cosmic inflation, the wavelength of GWs is smaller than the Hubble scale ("inside the horizon"). As inflation proceeds, the GW's wavelength redshifts and eventually is "ouside of the horizon", i.e. becomes larger than the Hubble scale (cf. Flanagan & Hughes, 2005, Sect. 5.2). A GW thus cannot be fundamentally characterised as such a "ripple on a background". Ultimately, there's only *one* metric, defying any *clear-cut* severing of "perturbations" from a "background". Rather, the picture of a ripple on a background is an approximate distinction, applicable only in certain regimes.

One should view the perturbative approach as describing the transition from a fundamental to a coarse-grained description (cf. Maggiore, 2008, Ch. 1.4.2). It's an effective field theory –

a tool convenient for approximations in a particular regime[16] up to a certain degree of accuracy; it's not a fundamental account.[17] (On a *fundamental* level, a GW is characterised via the Weyl tensor (for details, see e.g. Weinberg, 1972, pp. 145; Padmanabhan, 2010, pp. 263; pp. 403), i.e. the trace-free part of the Riemann tensor:

$$C_{abcd} = R_{abcd} - g_{a[c}R_{db]} + g_{b[c}R_{d]a} + \frac{1}{3}Rg_{a[c}g_{d]b}.$$

It encodes the purely gravitational degrees of freedom. It's constrained, but not determined by the matter distribution.)

This non-fundamentality needn't disconcert us. Applications of physical theories (almost) invariably operate on a non-fundamental conceptual level. In particular, effective field theories, including those with cut-offs, are omnipresent in physics (see, e.g., Gripaios, 2014). However, the present chapter is concerned with whether the standard arguments show that we should be committed to GW energy on a *fundamental* level. (This isn't to say that something interesting can – and should – be said about the *emergent* ontology of GW theory as an effective field theory, cf. Crowther, 2016; Read, 2017 for gravitational energy in GR as an emergent concept; Wayne, 2017 for GW theory as an effective field theory.)

So, what conclusions can be drawn regarding the perturbatively defined GW energy? Two difficulties obstruct a straightforward realism about it.

One arises, when $T_{ab}^{(\text{eff})}$ (or $t_{ab}^{(\text{eff})}$) is invoked *to explain* how the 2nd order perturbation propagates on the background. This suggests that the vacuum Einstein Equations in first and second order suffice to explain the behaviour of the *background* metric, whereas the behaviour of the *perturbations* calls for an explanans: it requires the GW energy-momentum as a source. One thus imputes to the perturbations a different status than the background. From a fundamental level of description, this explanatory asymmetry is indefensible: no perturbative order of the metric is privileged. Interpreting $T_{ab}^{(\text{eff})}$as a cause is tantamount to demanding an explanation for the *non-linear* terms of GR – an explanation that the *linear*

[16] The perturbative expansion in the so-called near-zone must be matched to a different kind of expansion in the so-called far-zone ("asymptotic matching"), cf. Maggiore, 2008, Ch. 5.1.6 This way boundary conditions at infinity can be incorporated, and divergences in the expansion can be avoided.

[17] The averaging over several wavelengths we prescribed above in order to overcome the gauge-dependence of the effective GW energy-momentum is in fact a special case of renormalisation group transformations, familiar from effective field theories to describe transitions from different levels of description (cf. Peskin & Schroeder, 1995, Ch. 12).

terms don't need. But why assume that? Doesn't this asymmetry just ignore the fact the GR is non-linear tout court (Aldrovandi, Pereira & Vu, 2007)? Why privilege the linear (1st order) parts of an *essentially* nonlinear theory?[18]

Another problem with the interpretation of the perturbative GW energy-momentum becomes evident in Isaacon's variational approach. For the correct field equations *both* the background metric and the perturbation must be treated as independent variables. Furthermore, the effective GW energy-momentum is defined variationally with respect to the background metric. This is exactly analogous to the definition the *matter* energy-momentum tensor, $T_{ab} = -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}L\right)$ with the matter Lagrangian L. The perturbation is thus effectively treated as a matter field. It's assigned its own energy-momentum tensor. Contrariwise, the background metric retains its spacetime interpretation. (Schutz & Ricci (2010, p. 41) are explicit about this.) But such an ontological re-categorisation of a (perturbative order of the) *spacetime* metric to an independent *matter* field looks like a momentous shift. It inaugurates an ontological difference that the perturbative approach per se doesn't sanction.

One might counter that this argument rests on a false dichotomy between spacetime and matter. Why shouldn't certain matter fields play the *spacetime role* (cf., for instance, Brown, 2007; Pitts, 2017; Knox, 2017b)*?* If they do, there is no reason why different matter fields shouldn't also play this role on different scales. I'm sympathetic to this line of reasoning. I surely don't deny that it's an *option.* But it mandates significantly more, independent arguments – well beyond what textbooks offer as arguments for GW energy.

Before moving on to practical shortcomings of the perturbative approach, I'd like to allay potential discomfort about the lack of general-covariance in the perturbative approach. In conversation, many physicists conceded a perceived oddity of the effective energy-momentum: it transforms tensorially only under transformations that leave the *background* metric invariant. It doesn't transform tensorially under more general transformations. GR simpliciter gainsays such a preferred status of a background. This "restricted" covariance is innocuous, though: not all sectors of GR's solutions must display the same transformation

[18] One should resist the temptation to compare GR with the linear Maxwell theory. A more illuminating analogy comes from comparison with the likewise genuinely non-linear vectorial Yang-Mills-type theories (cf. Deser, 1970). It's noteworthy that in such Yang-Mills theories energy is localisable: thus, the problems with localising gravitational energy in GR does *not* originate in its nonlinearity *per se*, as is sometimes claimed.

symmetries (cf. Belot, 2011, esp. pp. 2870). This is the case here. The boundary conditions for gravitationally radiating systems impose special symmetries on the resulting solutions. These symmetries here are reflected in the special status of the background metric.

I now conclude this look at the perturbative approach with two of its practical shortcomings. They considerably lessen its value for *any* applications of GW energy (Poisson & Will, 2014, Ch. 12.2.5). Firstly, the perturbative approach affords no notion of a "gravitational energy of the system". (The quotation marks here flag that I refrain from any ontological commitment to the *formal-mathematical* terms, thus labelled. This neutrality permits me to acknowledge that these terms appear in calculations, without having to promote them to elements of our ontology.[19]) Consequently, the perturbative approach falls short of paradigmatic applications of GW astrophysics, e.g. binary systems, whose "gravitational energy" decreases, when emitting GWs. In the same vein, a perturbatively defined GW energy is also too crude to deliver the "flux of angular momentum". The latter is important for the correct description of millisecond pulsars. Their rotation rate increases due to a transfer of "angular momentum" from the accretion disk surrounding the pulsar (Poisson & Will, 2014, Ch. 12.2.4). As the primary *raison d'être* of defining GW energy consists in its astrophysical utility, the perturbative road thus seems like a blind alley.

In summary, we pointed out three difficulties for accommodating perturbatively defined GW energy within GR's fundamental ontology and ideology. Firstly, as a source term for the propagation of the metric perturbations, it introduces an explanatory asymmetry between linear and nonlinear terms. This isn't licenced for GR as an essentially nonlinear theory. Secondly, it creates an ontological (spacetime vs. matter) asymmetry amongst perturbative orders. On some (prima facie plausible) views about spacetime, this asymmetry may seem drastic. Thirdly, the perturbative approach as a *whole* is unsuitable for astrophysical applications. This renders a perturbatively defined GW energy-momentum a questionable starting point for both physics and conceptual analysis.

[19] This situation is analogous to the quantum potential in Bohmian Mechanics. It's controversial that one should include it as part of the physical ontology – rather than, say, a manifestation of the "Aristotelian" inertial structure of Bohmian Mechanics (cf. Goldstein, 2017, sect. 5).

II.2.4 The Noetherian perspective

The most systematic approach to GW energy-momentum is via (a suitable generalisation of) Noether's Theorem. It treats GR's metric like a garden-variety classical field. Subsequently, I'll examine whether the Noetherian perspective by itself vindicates the ascription of energy to GWs. The discussion commences with a more general consideration of Noether's Theorem in GR. This enables us to grasp both the specific problems of GW energy, as well as those of gravitational energy more generally.

From a gauge-theoretic point of view (broadly construed), one of GR's characteristics is its general covariance: it's invariant under general diffeomorphisms. General covariance as a local symmetry of the Einstein-Hilbert action, $\int d^4x \sqrt{|g|}R$, allows for an application of Noether's 2nd Theorem. The latter links local gauge symmetries and conserved quantities (for technical and conceptual details, see Maggiore, 2008, Ch. 2; Brading & Brown, 2000; Brown & Brading, 2002).

For applications in GR, two reasons commend a less known generalisation of Noether's Theorem: gauge-dependence and the restrictive nature of some assumptions underlying Noether Theorem, respectively.

Firstly, the results of Noether's Theorem are gauge-dependent. Laxness regarding gauge-dependence wreaked considerable havoc in the history of GW theory (Kennefick, 2007, Ch. 4,5). In that light, it's imperative to be particularly circumspect in ensuring the gauge-invariance of any result. A related drawback of Noether's Theorem is that it only exploits the information encoded in the vanishing of the interior contributions of the varied action. This seems unduly restrictive: why assume *a priori* that GR's metric suitably "flattens out" at infinity so that contributions to the boundary can be discarded without impunity? The Klein-Utiyama Boundary Theorem addresses both issues (cf. Brading & Brown, 2000); Ohanian, 2013, Appendix 1 recapitulates the technical details for the GR case).

Consider the action $S[\psi_i] = \int d^4x \mathfrak{L}\,(\psi_i, \partial\psi_i, x)$ of the generic fields ψ_i. It's assumed to be invariant (up to a surface term) under an infinite-dimensional Lie group $G_{\infty,\rho}$ of transformations which smoothly depend on ϱ functions $p_\alpha(x^a)$ and their derivatives[20]

[20] For simplicity, a restriction is made to first derivatives.

$\partial_\beta p_\alpha(x^a)$, and which give rise to the variation of the dynamical fields ψ_i (of generic tensorial type), $\delta\psi_i = \sum_\alpha (a_{\alpha i}\Delta p_\alpha + b^c_{\alpha i}\partial_c \Delta p_\alpha)$. Here, $a_{\alpha i}$ and $b^\mu_{\alpha i}$ are coefficient functions that depend on x^a, ψ_i and $\partial_c\psi_i$. (The Δp_α's indicate that we are taking infinitesimal p_αs.)

Then, according to the Klein-Utiyama Theorem, there exist three sets of ϱ relationships:

- $\sum_i \; a_{\alpha i}\frac{\delta\mathfrak{L}}{\delta\psi_i} \equiv -\sum_i \partial_c\left(a_{\alpha i}\frac{\partial\mathfrak{L}}{\partial(\partial_c\psi_i)}\right)$
- $\sum_i \; b^\mu_{\alpha i}\frac{\delta\mathfrak{L}}{\delta\psi_i} \equiv -\sum_i\left(a_{\alpha i}\frac{\partial\mathfrak{L}}{\partial(\partial_\mu\psi_i)} + \partial_\nu\left(b^\mu_{\alpha i}\frac{\partial\mathfrak{L}}{\partial(\partial_\nu\psi_i)}\right)\right)$
- $\sum_i\left(b^\nu_{\alpha i}\frac{\partial\mathfrak{L}}{\partial(\partial_\mu\psi_i)} + b^\mu_{\alpha i}\frac{\partial\mathfrak{L}}{\partial(\partial_\nu\psi_i)}\right) \equiv 0.$

Here $\frac{\delta\mathfrak{L}}{\delta\psi_i}$ denotes the variational derivatives with respect to ψ_i, i.e. the familiar Euler-Lagrange expressions for ψ_i. Germane to our purposes is the first identity. After some rearranging, one can infer from it that the Noetherian 4-current

$$j^\mu_k := -\sum\nolimits_i \left\{\frac{\partial\mathfrak{L}}{\partial\big(\partial_\mu\psi_i\big)}\frac{\partial(\delta_0\psi_i)}{\partial(\Delta p_k)} + +\mathfrak{L}\frac{\partial(\delta x^\mu)}{\partial(\Delta p_k)} - \frac{\partial(\Delta\Lambda^\mu)}{\partial(\Delta p_k)}\right\},$$

(with the terms Λ^μ, arising, when the action isn't strictly invariant, such as in the case of GR) can be brought into the following form:

$$j^\mu_k = b^\mu_{ki}\left(\frac{\partial\mathfrak{L}}{\partial\psi_i} - \partial_\nu\frac{\partial\mathfrak{L}}{\partial\big(\partial_\mu\psi_i\big)}\right) + \partial_\nu U^{[\mu\nu]}_k.$$

The last term, $U^{[\mu\nu]}_k$, is a so-called superpotential, antisymmetric in its upper indices (for details see Trautmann, 1962).

Let's now apply the Klein-Utiyama Theorem to GR with the truncated ("ΓΓ") Lagrangian

$$\bar{\mathcal{L}} = 2g^{ab}\Gamma^d_{a[b}\Gamma^c_{c]d}.$$

(It's dynamically equivalent to the Einstein-Hilbert Lagrangian, see, e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 19.19.)

The last two identities of the Klein-Utiyama Theorem have some interesting implications for the form of the superpotential, as well as for the mutual constraints of the Einstein Equations and the matter field equations on each other, respectively (Ohanian, 2013, Appendix 1; Brown & Brading, 2002, sect. IV).

The first identity of the Klein-Utiyama Theorem, together with the Einstein Equation, entails:

$$\sqrt{|g|}(T_a^b + t_a^{\ b}) = \partial_c \mathfrak{W}_a^{[bc]}.$$

Here $t_a^{\ b} := \frac{1}{\sqrt{|g|}}\left(-\bar{\mathcal{L}}\delta_a^b + \frac{\partial \bar{\mathcal{L}}}{\partial(\partial_b g_{de})}\partial_a g_{de}\right)$ denotes the so-called Einstein pseudotensor (see Dirac, 1975, Ch. 32 for a handier form) and a superpotential $\mathfrak{W}_a^{[bc]} := g_{ae}\partial_d\left(|g| g^{e[b} g^{c]d}\right)$, again antisymmetric in its upper indices.

The Einstein pseudotensor corresponds to canonical energy-momentum associated with g_{ab}, as one would expect it from other field theories. The metric g_{ab} can be regarded (e.g. by Maggiore (2008, Ch. 2.1)) as a rank-2 matter field on Minkowski space. This suggests that $\mathfrak{T}_a^b := T_a^b + t_a^{\ b}$ should be interpreted as the “total” (matter plus gravitational) energy-momentum.

Thanks to the superpotential’s antisymmetry in its upper indices, it obeys a continuity equation:

$$\partial_b\left(\sqrt{|g|}\mathfrak{T}_a^b\right) = \partial_{b,c}\mathfrak{W}_a^{[bc]} \equiv 0.$$

Albeit *not* a tensor equation, the continuity equation holds in *every* coordinate system (Schrödinger, 1950, p. 104). It’s tempting to construe it as a conservation principle, reflecting the absence of sinks and sources of the total energy-momentum flux.

Let’s for the moment ignore potential technical problems, related to the convergence of integrals. Furthermore, suppose that we can interpret the Einstein pseudotensor as canonical energy-momentum of the metric. (We’ll regard the latter as representing the gravitational field.) Then, by culling from the pseudotensor those degrees of freedom associated with GWs, we get the GW energy-momentum: select from canonical energy-momentum associated with the whole spacetime (“total gravitational energy-momentum”) those “radiative” contributions identifiable with the GW. This is the standard Noetherian approach to GW energy (see Brading, 2005 for historical details).

Drawing on the discussion in §2.3, one immediately spots a problem of this approach: It relies on a clear separation of degrees of freedom associated with the GW, and those of the background. Such a separation isn't possible for arbitrary perturbative orders (or even at all for most realistic spacetimes).

Yet a graver issue besets the Noetherian GW energy. It generalises to the question of the status of gravitational energy in GR: should we interpret the resulting pseudotensor realistically? The question encompasses two aspects. One concerns problems of the interpretation of the Noetherian energy-momentum 4-currents ("*local* energy-momentum"); the other concerns the question whether these 4-currents give rise to well-defined "*global*" (integral) quantities.

Let's first focus on local gravitational energy-momentum. Three major difficulties encumber a straightforward realist interpretation of the Einstein pseudotensor as gravitational energy (see Dürr, forth. for a detailed analysis, i.e. Ch. III of this thesis): 1. Gauge-dependence, 2. Index-nonsymmetry, and 3. Ambiguity.

> 1. As a pseudotensor, the Einstein pseudotensor doesn't transform tensorially under arbitrary coordinate transformations. It's invariant only under linear (affine) transformations. However, in spacetimes other than Minkowski's, linear transformations are no longer privileged. (In terms of a Kleinian approach to geometry (for details, see Wallace, 2016): in general, the pseudotensors' invariance group won't coincide with the symmetry group of the spacetime on which they live.)
>
> The object denoted by the pseudotensor thus depends on the *conventional* preference of certain coordinate system – in contrast to the invariance one would naturally demand of a physical quantity (cf. Vollmer, 2010).
>
> Weyl (1923, p, 273) neatly summarises the dilemma: "Indeed all the [pseudotensor components] can, through a suitable choice of a coordinate system, be made to vanish; [...] on the other hand one obtains [pseudotensor components] that are different from zero in a 'Euclidean', completely gravity-free world by using a curvilinear coordinate system, where it seems pointless to speak of gravitational energy."
>
> 2. Einstein's pseudotensor isn't symmetric in its indices. This compromises its physical suitability for defining angular momentum.
>
> One can take care of the asymmetry by dint of the Belinfante-Rosenfeld technique. But the latter favours the Poincaré group in a manner prima facie not justified within GR (Leclerc, 2006).

3. Ambiguity: The canonical energy-momentum 4-current above is defined only up to a choice of a superpotential. In itself, none is less apt than other choices. The pseudotensor thus is vastly underdetermined: In fact, (uncountably) infinitely many choices are possible (Bergmann, 1958; Komar, 1959). Different pseudotensors are known that yield different energy-momentum distributions for certain spacetimes (Virbahdra, 1990). (This doesn't seem to be the rule, though.)

Vis-à-vis such difficulties, should one perhaps follow Weyl's suggestion that a realist interpretation of the Klein-Utiyama-Noetherian results be reserved only for the *global/integral* quantities, associated with the 4-currents?[21]

For the integrals to be well-defined, however, the metric must satisfy certain conditions at infinity ("asymptotic flatness"). How to formulate the required flattening-out in a coordinate-independent way? This leads to so-called conformal techniques (e.g. Geroch, 2013, Ch. 35-38; for details, see Frauendiener, 2004). The rather involved details shan't detain us here. Roughly speaking, one augments the spacetime $(\mathcal{M}, g)$ by boundary points,"∞", corresponding to idealised end-points at infinity along time- or null-like geodesics. By a suitable choice of a smooth scale factor, $\Omega: \mathcal{M} \to \mathbb{R}^{+}$, one can shrink lengths (encoded in the "conformal" metric $\tilde{g} := \Omega \mathrm{g}$) such that infinity can be represented as points on the compact augmented manifold, $(\mathcal{M} \cup \infty, \tilde{g})$. At infinity, Ω is assumed to vanish. Asymptotic flatness in this pictures then is essentially defined as the requirement that in a neighbourhood of infinity the Ricci tensor $\tilde{R}_{ab}$ associated with $\tilde{g}$ vanish.

Asymptotic flatness elicits numerous delicate questions: how restrictive are the constraints it imposes on the space of formally admissible spacetimes? That is: How typical are asymptotically flat solutions in the space of all solutions? How physically robust is asymptotic flatness? That is: How stable is an asymptotically flat spacetime against perturbations in the boundary conditions? To what extent are realistic spacetimes asymptotically flat? How are the different notions of asymptotic flatness at space-like, null or time-like infinity related? How formally plausible is the above Noetherian gravitational energy-momentum definable on

[21] Weyl (1923, p. 273; cf. Schrödinger, 1950, p. 100) writes: "Still, from a physical point of view, it seems meaningless to introduce the [pseudotensor] as energy components of the gravitational field, for they form neither a tensor nor are they symmetric [...]. Even if the differential relations [i.e. the pseudotensor-based continuity equation, P.D.] are without any real physical meaning, they do give rise to an invariant statement of conservation via integrating over an isolated system."

asymptotically flat spacetimes? How is it related to alternative proposals, such as the Bondi mass?

These questions lie beyond the present chapter's ambit. For our purposes, the first three are primarily relevant. It's clear that asymptotically flat spacetimes are exceptional amongst formally possible ones: any deviation from Ricci-flatness at infinity mars asymptotic flatness. For some objects, one can approximate their ambient spacetime to a good approximation as asymptotically flat. (Recall the caveats from §2.2, though.) But – at the very least – on cosmological scales, the asymptotically flat approximation of realistic, astrophysical objects breaks down: within a certain regime, one can – for reasons of computational convenience – at best *embed* a (relevant) patch of their physical spacetime into an asymptotically flat one. As argued in §2.2, it's an idealisation in Norton's sense: the embedding spacetime is an unrealistic, surrogate spacetime. Consequently, realism about notions of gravitational energy based on asymptotic flatness isn't straightforward.

In conclusion, the Noether Theorem provides a systematic route to "gravitational" and "GW energy". (The apostrophes again flag that the terms refer to the *mathematical* objects, conventionally thus labelled.) The result is both a local and global notion. The former is unconvincing. Global notions, on the other hand, prompt tricky questions. Some answers can be given, e.g. regarding the positivity of "gravitational energy" or the "energy-momentum flux of GWs" (for this impressive, beautiful theorem, see e.g. Straumann, 2013, Ch. 3.7, 6.1). Even so, the conditions that their definition requires are idealisations in Norton's sense. This casts into doubt a *naive* realism about them.

Given this controversial status of gravitational and GW energy, it seems desirable to eschew reference to it, altogether. I demonstrate that this is feasible for the explanation of the orbital decay of double pulsars.

II.3 Aren't binary systems evidence for GW energy?

In this section, I analyse and critique the received account of binary systems. I proffer an alternative explanation exclusively in terms of the general-relativistic equations of motion and the Einstein Equations. It's argued to be superior to the standard account.

III.3.1 The standard view

On the standard interpretation of the binary problem, one explains the orbital decay via the system's total energy being carried away by the emitted GWs.

More precisely, the standard interpretation, as presented in the astrophysics literature (e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 18.8; Poisson & Will, Ch. 6-12), starts from an energy balance of the (quasi-symbolic) form $\dot{E} = -L$. The energy-flux on the r.h.s. -the energy-momentum radiated away- compensates the change in the system's total energy-momentum on the l.h.s.

Such an energy balance is provided by a realist interpretation of the continuity equation for total energy-momentum, based on the energy-momentum tensor of matter and a pseudotensor, as encountered in §2.4. A standard choice in astrophysics is Landau and Lifshitz's index-symmetric pseudotensor (for details, see Landau & Lifshitz, 1971, §101). It satisfies the continuity equation

$$\partial_b \left(|g| \left(T^{ab} + t^{ab}_{(LL)} \right) \right) = 0.$$

Integration over a 3-region $\mathcal{V}_3$ yields the corresponding total energy-momentum $P^a = \left(E/c\,, P^i\right)$, contained in this volume:

$$P^a = \frac{1}{c} \int_{\mathcal{V}_3} d^3x |g| \left(T^{a0} + t^{a0}_{(LL)} \right)$$

(If one choses $\mathcal{V}_3$ to be infinite in an asymptotically flat spacetime, this total energy-momentum transforms like a Minkowski vector (density) at infinity.) Sometimes, it's convenient to express the volume integrals as surface integrals. Exploiting that the continuity equation can be re-written in terms of Landau and Lifshitz's superpotential $H^{\alpha\mu\beta\nu}$, one obtains

$$P^a = \frac{c^3}{16\pi G} \oint_{\partial \mathcal{V}_3} d\sigma_k \partial_\mu H^{\alpha\mu 0k}.$$

For the total energy of the system (i.e. matter *cum* gravitational), we take the volume to lie in the proximity of the source, i.e. close to the so-called near-zone. Here, "close" means "not many wavelengths away from the source". That is: The distance is at most comparable to the characteristic wavelength of the emitted gravitational radiation. The total energy is then determined by extracting those degrees of freedom associated with the matter and non-radiative gravitational energy-momentum:

$$E = \left[\frac{c^4}{16\pi G}\oint_{\partial \mathcal{V}_3^{NZ}} d\sigma_k \partial_\mu H^{\alpha\mu 0k}\right]_{cons}.$$

The brackets "$[\dots]_{cons}$" indicate that one removes those degrees of freedom associated with gravitational radiation. One is then left with what formally corresponds degrees of freedom of a conservative system. This energy E decreases, when the system emits GWs.

The energy loss is the energy that the GWs allegedly carry away. One can determine it by evaluating Landau-Lifshitz's above integral far away from the source, the so-called wave-zone. There, the gravitational degrees of freedom associated with radiation dominate.

Two remarks on this procedure are in order. Firstly, in order to actually perform the above evaluations, one generally needs different approximation schemes of the chosen model of the radiating source for different domains/"zones" (hence the above superscripts above the integration volumes). They are glued together via a suitable matching technique. Secondly, I take the standard explanation of the binary pulsars to invest the alleged energy transport via GWs with *explanatory clout*: the system loses energy *because* GWs carry away energy. This in turn explains why the orbits of the binaries decrease. More on this shortly.

For the sake of concreteness, consider a system of two point-particles of equal mass *M*, rotating around their centre point with constant angular velocity ω and with the coordinate distance *a* between them (see e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 18.8). The corresponding balance equation in leading order is:

$$\frac{d}{dt}\left(\frac{1}{2}(2M)v^2 - \frac{GM}{2a}\right) = -\frac{128G}{5c^5}M^2a^4\omega^6.$$

The l.h.s. describes the system's change in total energy. (For higher accuracy, next-leading order terms can also be consistently incorporated. This yields correction terms to the Kepler potential (responsible e.g. for the perihelion shift), as well as velocity-dependent terms, construed from a Newtonian perspective as corrections to kinetic energy.) The r.h.s. is interpreted as the energy loss via emission of GWs. From this balance equation, one can derive an equation for the spin-up, i.e. the rate of change of the orbital period *P*, directly accessible to observation:

$$\dot{P} = -\frac{96}{5} 4^{\frac{1}{3}} \pi \left(\frac{2\pi G M}{P} \right)^{5/3}.$$

Via the following three propositions we can explicate the logical structure of the standard explanation:

(1) Along the bound or scattered orbits of a 2-body system (i.e. if the orbits prescribed by the equations of motion don't decay), the system's total (i.e. gravitational *cum* matter) energy-momentum is conserved.

(2) The binary system's total energy-momentum is conserved.

(3) GWs carry away energy-momentum from the system.

Given (1), it follows from (2) and (3) that orbits decay: *because* GWs carry energy away from the system, the *otherwise bound* orbits decay. GW energy in (3), alongside with the principle (1), subserves as the explanans for the decrease in total energy. Via modus tollens of (1) it entails the explanandum, the orbital decay ("$\mathcal{E}$"):

$$(3) \longrightarrow \neg(2) \overset{(1)}{\rightarrow} \mathcal{E}.$$

II.3.2 Criticism

Three types of defects afflict this standard explanation. It contains assumptions and concepts fundamentally at odds with GR. Furthermore, two of its premises turn out to be unnecessarily strong, and in fact unjustified, respectively.

The first strand of criticism applies to all three steps: (1), (2) and (3) involve GW energy or gravitational energy. Both are problematic notions (cf., for instance, Hoefer, 2000, Curiel, 2000, Petkov, 2017). What is more, the standard explanation tacitly assumes that one can clearly *sever* the system's gravitational energy-momentum from that of the GW. Whilst true for the lowest perturbative orders, in higher orders the radiative and the static gravitational degrees of freedom are inextricably interwoven. In consequence, the equations of motion for the binaries defy a standard Lagrangian or Hamiltonian formulation. (Recall that in Classical Mechanics systems with generic (not purely velocity-dependent) friction can't be treated within the standard Lagrangian framework. In leading order, one can in fact regard the back reaction on binaries due to gravitational radiation as a form of friction. This is done e.g. in. Padmanabhan (2010), Ch. 9.6.2.) Hence, the system's total energy-momentum can't even be defined for higher perturbative orders.

To introduce energy for (gravitationally or electromagnetically) radiating systems, one must appeal to a principle of energy-momentum conservation: The radiated energy is constructed *such that* energy conservation is restored.[22] But in GR this principle becomes doubtful. We saw this in §2.3: Global conservation of matter energy-momentum no longer holds except in symmetric spacetimes. Likewise, in §2.4 we discussed the problems that impede a realist stance towards the local continuity equation $\partial_\mu\left(|g|\left(T^{\mu\nu}+t^{\mu\nu}_{(LL)}\right)\right)=0$.

A second line of attack aims at assumption (3) as gratuitously strong: it's unnecessary to demand that energy be *carried away.* For the argument to go through, it suffices that the system's total energy-momentum decreases. The loss in energy-momentum needn't be compensated by equally real energy, ascribable to the GW and transported continuously from one place to another. What matters is the *energy loss* - not a story about how to track the "missing" energy.

What's the difference between violation of energy conservation and the "missing energy" being carried elsewhere? After all, as Curiel (2000, p. 9) pithily remarks: "One cannot tag hunks of energy as one can hunks of cheese, and so one cannot identify the energy that this system lost with the energy that that one gained in the same way one could if one were talking about cheese."

Three considerations bear upon the choice between failure of energy conservation and energy transfer: 1. the contingency of energy conservation on symmetries, 2. the existence of a satisfactory formal account/representation of the energy transport, and 3. the explanatory value of postulating energy transport rather than energy decrease simpliciter, respectively.

In GR the vanishing covariant divergence of the energy-momentum flux in a direction of a time-like $\xi, \nabla_a(T^a_b\xi^b)\neq 0$ doesn't yield a conserved global/integral quantity. It's widely acknowledged that this simply reflects that energy conservation no longer holds - a feature less revisionary than may appear at first sight: after all, the special-relativistic conservation laws for energy-momentum and angular momentum depend on the 10 Killing vectors of Minkowski space. Generic GR spacetimes, by contrast, lack any symmetries (§2.3). So, absent such symmetries, why expect energy-momentum conservation to hold?

[22] Poisson & Will (2014, Ch. 12.1) stress the analogous procedure for electromagnetism.

Secondly, there doesn't exist any conceptually unproblematic way to *express* the dissipated energy. The most prominent way, for instance, via pseudotensors, faces numerous challenges, reviewed in §2.4. By contrast, the energy emitted by an electromagnetically radiating system allows for a tensorial representation. Indeed, there is a large consensus regarding the "non-localisability" of the dissipated energy (expressed, for example, in Misner, Thorne & Wheeler, 1973, p. 467; cf. Curiel, 2013 for a rigorous proof): it's indeed impossible to specify *where* in spacetime this GW energy resides. No argument, however, is presented why one should believe in the *existence* of gravitational/GW energy to begin with. Such ominous non-localisability poses interpretative challenges. (At the minimum, it forces us to revise established conceptual frameworks for field theories, such as Anderson's (1967) framework of geometric objects. Anderson explicitly states that (pseudotensorial) gravitational energy doesn't form a geometric object.) Hence, it seems desirable to eschew non-localisable quantities, if possible. We'll explore this option presently.

A third aspect germane to adjudicating between energy non-conservation and energy transport via GWs is the explanatory surplus value of the latter choice: does postulating GW energy transport help us better explain, or understand, certain phenomena? If that were the case, an inference to the best explanation would significantly strengthen the claim that a system's energy-momentum non-conservation should be accounted for in terms of GW energy-momentum transport.

I next turn to this question. Is a better explanation of the binary pulsars' orbital decay available? I submit this is the case. (Applications of gravitational energy in other contexts, e.g. merger scenarios of Black Holes or the intricate interaction of GWs with a detector, such as LIGO, call for an independent study. My subsequent analysis, confined to binary pulsars, thus has more *programmatic* than conclusive character.)

II.3.3. A dynamical explanation

Let's revert to premise (1), pivotal to the standard explanation. Why believe that if the particles follow scattered or bound orbits, the system's total energy is conserved?

The answer one *would like* to give is: because the theory's dynamics – i.e. the equations of motion (EoMs)[23] together with the Einstein Equations (EEs) – tell us so. In *Newtonian* Mechanics, energy is generally conserved; the EoMs imply that orbits of celestial bodies are either hyperbolic ("scattered") or elliptical ("bound"). A planet or comet cannot spiral into the sun without energy dissipation, e.g. via "tidal friction". (The latter would count as the *cause* for the planet's orbital decay.)

Why assume the same *in GR*? I propose that we simply shouldn't. GR's EoMs and EEs dictate that the binary pulsars' orbit decay. There is no need to invoke any quantity to explain the *deviation* from bound orbits. If one were to maintain (1), one would pick out from the *full* EoMs those parts that describe conservative systems, i.e. systems whose energy is conserved. The deviation from the orbits of these conserved systems would then be explained in terms of the energy losses via GWs. One would split the EoMs into a conservative ("cEoMs") and a non-conservative part; the former would be treated as explanatorily distinguished. The deviation of the system's actual orbits from the ones obtained from the cEoMs would call for an explanation in terms of energy losses, whereas the conservative orbits would be seen as the explanatorily default motion. What vindicates such a split? The reason seems to be little more than habituation from pre-GR physics (where energy-momentum conservation is, of course, valid and ubiquitously useful). The conviction that all systems are conservative, unless some friction or radiation dissipates energy, is so deeply engrained in our physical hunches that it has ossified into dogma. But if its plausibility presupposes the validity of energy-momentum conservation, and the latter needs to be jettisoned (or at least becomes controversial) in GR, the justification of (1) lapses.

Instead, GR compels us just to accept that no bound solutions of the general-relativistic binary problem exist (Papapetrou, 1957; 1958). As Cooperstock and Tieu (2012, p. 83) put it, "on this basis, the period-changing binary pulsar is simply manifesting its conformity with the mathematical demands of Einstein's General Relativity rather than the preconceptions regarding energy."

It thus seems apposite to supplant (1) by the following principle:

[23] By the EoMs I mean the field equations for the *non-gravitational*/*matter* fields.

(1_{GR}) In GR, two-body systems emit GWs and their orbits are in-spiralling.

By the same token, let's revise (2) by embracing GR's failure of energy-momentum conservation. In order to evade the problems associated with gravitational energy, we must also modify the reference to *total* energy-momentum. As a result, let's supplant (2) by

(2_{GR}) In generic (non-static) spacetimes, matter energy-momentum isn't conserved.

Since (3) turned out to be unnecessarily strong and involved the controversial existence of GW energy, we drop it.

Both (1_{GR}) and (2_{GR}) needn't be posited as independent principles: They ensue from the EEs and EoMs. The conjunction of the EEs and the EoMs thus furnishes us with an explanation for the orbital decay (as before: "$\mathcal{E}$"): *because* of the EEs and the EoMs, the orbits of the system are inspiralling.

Let's be a bit more precise and specify the antecedent matter conditions (QUA) that -due to the EEs and the EoMs- result in inspiralling orbits of systems emitting GWs. For a system to generate GWs, its matter quadrupole moment tensor

$$I^{ij} = \int d^3\, y T^{00}(ct, \vec{y}) y^i y^j$$

must vary in time.

We then arrive at an explanation that makes reference only to a condition on the matter energy-momentum (QUA), the EEs and the EoMs. It manifestly draws only on bona fide GR concepts and assumptions. In lieu of GW energy and (1) in the standard explanation, the explanantia in the dynamical explanation are the EEs and EoMs, as well as the time-varying quadrupole moment of the matter energy-momentum distribution as the antecedent conditions. *Because* the quadrupole moment of the matter energy-momentum distribution varies in time (QUA), the EEs and EoMs imply that the system's orbits decay:

$$\text{(QUA)} \xrightarrow{(EoMs)\&(EEs)} \mathcal{E}.$$

I'll dub this the "dynamical" explanation of the orbital decay, as its explanantia are GR's *dynamics* – the general-relativistic EoMs and EEs.

Interestingly, according to this dynamical interpretation, GWs *no longer* play any explanatory role. Rather, the emission of GWs and the orbital decay share a common cause in the antecedent matter condition:

$$(\mathrm{QUA}) \xrightarrow{(EoMs)\&(EEs)} \mathcal{E}\ \&\ \mathrm{GW\ emission}.$$

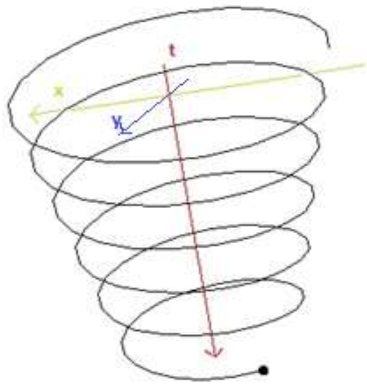
By means of illustrating the dynamical explanation, it's insightful to address Petkov's recent objection to the standard interpretation of binary systems.[24] According to Petkov (2017, p. 11), "[…] [it] contradicts general relativity, particularly the geodesic hypothesis […], because by the geodesic hypothesis the neutron stars, whose worldlines had been regarded as exact geodesics […] *move by inertia without losing energy since the very essence of inertial motion is motion without any loss of energy.*"

(For the sake of concision, I'll ignore mathematical qualms about the use of point-particles in GR, cf., however, Straumann, 2013, Ch. 6.4, 6.5; Wayne, 2017.) The dynamical explanation resolves the conflict Petkov animadverts upon. With respect to the metric that satisfies the EEs (determined, say, numerically to the desired degree of accuracy), the pulsars indeed follow geodesics. (For dust point-particles, as which the binaries are modelled with mass *m,* worldline $z^a(\tau)$ and the energy-momentum tensor $T^{ab}(x) = \frac{m}{\sqrt{|g|}} \int z^a(\tau) z^b(\tau)\, \delta^4(x - z(\tau))d\tau$, the EoMs needn't be postulated independently: the EEs imply them.). In contrast to the rectilinear ones in Minkowski spacetime, these geodesics are spiraling-in towards each other. Four-dimensionally, their wordlines can be envisioned as a double helix whose radius decreases with time. This helical shape of the binaries' worldlines is a brute fact of GR's spacetime geometry (more precisely: GR's inertial structure, encoded in the affine connection compatible with the metric that satisfies the EEs). The orbits of the 2-body problem -i.e. the sequence of three-dimensionals projections of the two worldlines onto simultaneity planes-just fail to be stable.

[24] Bondi (as reported in Kennefick, 2007, p. 200) voiced the same argument earlier.
It's not clear, however, that Bondi always distinguishes clearly enough between the emission of GWs and the transfer of energy-momentum via GWs. I take the novelty of Petkov's argument to consist in making this distinction.

Fig. 3:

Helical worldline of one of the binary partners. One spatial dimension is suppressed.



The dynamical explanation desists from identifying the system's total energy-momentum with the "energy-momentum" constructible from the conservative parts of the dynamics.[25] It dispenses with any notion of energy-momentum other than the energy-momentum of matter. (The latter generically turns out not to be conserved.) For the point-particles in free-fall this energy coincides with their rest mass-energy. It's indeed conserved. In summary, Petkov's diagnosis of inconsistency of the standard explanation rests on the (tacit) premise that the orbits of the general-relativistic 2-body problem must be bound and that the inspiralling must be explained in terms of energy-momentum *transport*.[26] The dynamical explanation rejects these assumptions as unwarranted.

II.3.3 Dynamical vs. standard interpretation

We are now in a position to compare the standard and the dynamical explanation. How does the latter fare vis-à-vis the former? I'll argue below that the dynamical account is superior on grounds of parsimony, universal scope, depth and unificatory power. The merits of the standard explanation are limited to familiarity to pre-GR concepts and principles.

For reasons of space, my discussion focuses on the binary system's problem of motion. I set aside the question whether there are other phenomena whose explanation involve GW energy. (Such might include the so-called CFS instability of rotating neutron stars (see e.g. Ricci

[25] More precisely, on the standard explanation, one would define the system's energy as the energy associated with the Einstein-Infeld-Hoffmann Lagrangian (see e.g. Straumann, 2013, Ch. 6.5).

[26] While Petkov (2012, p. 136), too, disputes that GWs transport away the energy, he suggests an alternative *causal* story in terms of tidal friction – a proposal that strikes me as ad-hoc.

& Schutz, 2010, sect. 6.2), (classical) black hole thermodynamics or astrophysical energy extraction processes (see Geroch, 1973).)

The standard explanation relies on gravitational energy, GW energy and energy-momentum conservation. All three are controversial concepts within GR's fundamental framework. Here, I'll refrain from the strong claim that such concepts are illegitimate within GR. Yet, given their controversial nature, it seems judicious to prefer explanations that do without them.

For a fairer comparison, let's first re-formulate and refine the standard explanation for orbital decay by explicating the more fundamental principles on which its elements rest. Those comprise:

- a time-varying quadrupole momentum distribution of matter (QUA), as in the dynamical explanation
- the Einstein Equations (EEs)
- the split ("Δ") of the EoMs into the conservative parts (cEoMs), associated with the dynamics of the system, and non-conservative parts, associated with the dissipative gravitational radiation.

From these building blocks, the three assumptions (1)-(3) of the standard interpretation can be reconstructed. A system's total energy-momentum is defined as the energy associated with the conservative dynamics encoded by cEoMs. By definition, it's conserved, unless the system radiates. Since the effective full EoMs also contain non-conservative dynamics (in other words: as a result of the cEoMs-EoMs split Δ), the GW depletes the system's energy.

On the now refined standard explanation, gravitational radiation is emitted -in accordance with the EEs and the full EoMs- because the system's quadrupole moment varies in time (QUA). Due to Δ, the solutions of the full EoMs deviate from those of the cEoMs; the system exhibits orbital decay. The refined standard explanation thus takes the following logical form:

$$\text{(QUA)} \xrightarrow{(EEs\&EoMs)} \text{GW emission} \overset{\Delta}{\rightarrow} \text{GWs deplete the system's energy} \xrightarrow{(cEoMs)} \mathcal{E}$$

The dynamical explanation, by contrast, took this form:

$$\text{(QUA)} \xrightarrow{(EoM)\&(EEs)} \mathcal{E}.$$

We can now squarely compare both. I first want to rebut three potential advantages one might see in the standard interpretation: its intuitive appeal and heuristic value, its close ties to observable quantities and its causal-mechanistic character, respectively.

With its appeal to familiar concepts and principles, the standard explanation of the binary problem has intuitive allure, and – especially in light of the analogy with electromagnetism-heuristic value.[27] Yet, many will dismiss such *subjectively* perceived advantages as irrelevant to an explanation's quality: does the (putative) un-intuitiveness of quantum mechanical explanations, say, of α-decay in terms of quantum tunneling lessen their value? Furthermore, our "intuitions" and "heuristic value" are considerably depend on the formal approach to the general-relativistic problem of motion which we adopt. Lehmkuhl (2017ab) distinguishes between what he calls the (more common) "*T*-approach" and the "vacuum approach", respectively. The former focuses on the energy-momentum tensor. It thus invites intuitions involving gravitational and GWs' energy-momentum. By contrast, the somewhat neglected vacuum approach focuses on the l.h.s. of the Einstein Equations. Owing to this different outlook, explanations involving energy – be it gravitational or non-gravitational- seem less natural in the vacuum approach.

A different line of defence of the standard view on binary systems might run as follows: Does the use of gravitational and GW energy perhaps enable us to ascertain more easily GR's empirical content? I fail to see how this could be true. If on the one hand one construes the argument as extolling the intuitiveness of the elements of the standard interpretation, it seems to collapse into the appeal to familiarity. Regarding that I have already voiced my misgivings. If on the other hand one construes the argument as the claim that the standard explanation renders the observational content more explicit, it's plainly false: information about the observables – here: the orbital decay – is no less present in the dynamical interpretation.

Perhaps most promising is the idea that the standard explanation provides a causal-mechanistic explanation. The guiding thoughts here are that firstly energy transfer is the hallmark of causal processes (see e.g. Dowe, 2009), and secondly that causal-mechanistic

[27] To be sure: The emphasis of such analogues between GR and non-GR physical theories played a vital *sociological* part in GR's reinvigoration and re-integration into mainstream physics in the 1950s and 60s (see Schutz, 2012 for details).

explanations are pre-eminent types of explanations. But this maneuver faces three objections. Firstly, the notion of causality – and *a fortiori* causal explanations – is notoriously ambiguous. In GR, the difficulties are even exacerbated (Curiel, 2000; 2015). Furthermore, it's not obvious to me that causal explanations *necessarily* involve energy transfer. Are the explanations of action-at-a-distance theories, say, Bohmian Mechanics or Feynman-Wheeler absorber theory (neither of which *prima facie* involves energy transfer) un-controversially non-causal? Assume for the sake of the argument that causality does necessarily involve energy transfer. Still, one can well question that causal-mechanistic explanations are *inherently* superior to non-causal ones (cf. Reutlinger & Saatsi, 2018). Electron degeneracy pressure, for example, is standardly explained in terms of Pauli's Exclusion Principle, i.e. a symmetry principle – rather than a causal mechanism. Why should this non-causal nature detract from the value of the standard explanation of, say, white dwarf formation in terms of the Pauli Principle?

In conclusion, the standard interpretation doesn't have any obvious *intrinsic* advantages. But perhaps its advantages are *comparative.* How do the standard and the dynamical explanation each score on the explanatory virtues of parsimony, scope and depth?

Regarding parsimony, note first that in terms of calculational efforts, both explanations are on a par. Regardless of which explanation one prefers, one has to solve the coupled set of partial differential equations formed by the EoMs and the EEs. Adherents of either explanation must use the same computational methods, e.g. approximation schemes.

The difference between both explanations thus boils down to: which status to attribute to the split Δ the (refined) standard explanation involves? As I argued earlier, there is no sound a priori justification for distinguishing the conservative parts of the EoMs. It's an additional postulate of the standard explanation. The dynamical explanation, by contrast, rejects it: it's committed neither to the split Δ nor the distinction of the cEoMs.

It also deserves reiterating here that the refined formulation of the standard explanation renders it transparent that claim (3), purporting the transport of GW energy-momentum, is explanatorily redundant. Only the decrease of the system's energy matters. Rather than contributing to the explanation of the orbital decay, (3) requires a substantive principle to hold: conservation of energy-momentum (EC) – more precisely, a realist interpretation of the formal energy-momentum balance of the pseudotensorial type, according to which the energy-momentum dissipated from a system continues to exist elsewhere:

$$\text{(QUA)} \xrightarrow{(EEq)} \text{GW emission} \xrightarrow{(cEoMs)\ \&\ \Delta} \text{GWs deplete the system's senergy} \xrightarrow{(EC)} (3).$$

Hence, if in order to support GW energy transport, one wants to appeal to the standard explanation as the best explanation, one must first substantiate not only the privileged status of the cEoMs, but also (EC) (a realist interpretation of the energy-momentum continuity equation). This need for two additional principles seems to undermine the prerequisite for the inference to the best explanation - namely that it be the *best* explanation qua its greater simplicity, all else being equal.

Last, not least, the dynamical account acquits itself particularly well in terms of parsimony, when matter only interacts gravitationally: then, the EEs imply the EoMs, simplifying the dynamical explanation to:

$$\text{(QUA)} \xrightarrow{(EEs)} \mathcal{E}$$

In summary, the verdict on parsimony *disfavors* the standard explanation: the latter employs more explanatory machinery (viz. energy conservation, gravitational energy, GW energy and their separation) than the dynamical explanation.

Perhaps the standard account's fortes lie elsewhere – for instance in scope: is the standard interpretation able to cover a wider domain than the dynamical interpretation? I dispute this, too. In higher perturbative orders, the gravitational and radiative degrees of freedom of the metric mix. It's no longer possible to unambiguously classify the higher order contributions of the EoMs as pertaining to the GW or the system (see e.g. Maggiore, 2007, p. 249, fn. 17) - as the standard interpretation via its split Δ presupposes. One could, of course, just stipulate by fiat that only the cEoMs describe the system qua their conservativeness. But such a decree lacks any physical foundation. In short, the scope of the standard explanation is limited to low levels of approximation (viz. 2.5 or 3PPN terms).[28] By contrast, the scope of the dynamical explanation coincides with the scope of GR and the general-relativistic EoMs: wherever classical GR is valid, a dynamical explanation of a phenomenon is possible – for instance, in

[28] In personal correspondence, Clifford Will stated: "In my opinion, a Lagrangian or a Hamiltonian for this problem only makes sense up to 2PN order, where energy is truly conserved (if you artificially turn off the 2.5PN radiation reaction terms, you can also write down a Lagrangian for the 3PN terms). Beyond this order, the fundamental things are the equations of motion."

non-asymptotically flat spacetimes, which don't permit the definition of any standard notion of gravitational energy.

This leads us to the issue of depth. At what level of description do the two explanations operate? From the aforesaid, it's clear that the (refined) standard explanation qua its reliance on the cEoMs and energy conservation employs non-fundamental principles. By contrast, the dynamical explanation only draws on the EoMs and the EEs. For non-quantum purposes, they may be regarded as fundamental. Hence, also with respect to depth, the dynamical explanation trumps the standard one. (I don't believe that depth, thus construed, is a value per se. It can be traded-off for other explanatory benefits (cf. Knox, 2016; 2017; Franklin & Knox 2017, ms). Ceteris paribus, however, it seems plausible to give preference to deep explanations.)

Lastly, let's turn to unificatory power. Does the standard account perhaps excel in this regard? Prima facie, one might think so: with its distinction of a conservative part of the dynamics -the cEoMs- it appears to preserve continuity with the dynamics of, and explanatory practices in pre-GR theories. Therefore, prima facie the standard interpretation's privileging of the cEoMs instantiates a subsumption under a general explanatory scheme, successful in pre-GR contexts.

The opposite is the case. Firstly, dividing the EoMs into a conservative and a non-conservative part isn't even consistently feasible. Moreover, in contrast to pre-GR theories, such a division of the EoMs is *artificial* in GR. Because of its reliance on this division, the standard explanation therefore cannot be subsumed under a more general explanatory scheme. The general explanatory scheme we find and apply also in other theories is just to take the EoMs -be they conservative or not; in pre-GR physics they just *happen* to be conservative- and determine from them how matter behaves under certain matter conditions ("MAT"). The unificatory explanatory scheme thus is:

$$(\mathrm{MAT}) \xrightarrow{(EoMs)} \text{explanandum.}$$

This is precisely the rationale of the dynamical explanation. The conservativeness of the EoMs is inessential.

From what I can see, this unificatory explanatory scheme is universally applied in other explanations in GR, as well. Think , for example, of the explanation of light bending in GR: light is deflected by gravitating bodies because (a) according to the EoMs of light, light rays follow null geodesics[29] and (b) the null geodesics linked with the metric that solves the EEs deviate from the straight geodesics of flat spacetime. (The explanations of perihelion shift, the Thirring-Lense effect or cosmic expansion of FLRW universes are equally straightforward.)

In conclusion, also regarding unification, the dynamical explanation trumps the standard one. The former instantiates a standard explanatory scheme of great unificatory power. By contrast, the standard account of binary systems implausibly generalises and elevates contingent features of pre-GR theories.

In summary, on four central virtues of good explanations – parsimony, depth, scope and unification – the dynamical interpretation of binary systems outperforms the standard one.

In this section, I presented an attractive alternative to the standard explanation of the binary system's orbital decay. It doesn't presuppose gravitational or GW energy, and proved superior on various criteria. Advantages of the standard account turned out to be specious. The section's title question can therefore be answered in the negative: the standard treatment of binary systems doesn't provide an inference to the best explanation for GW energy.

II.4 Outlook

For the line of thought pursued here, two directions of further enquiry are particularly promising:

One concerns an in-depth analysis of the status and role of the various proposals for gravitational energy-momentum in GR. Of particular interest here is the status of Bondi's News Function (e.g. Straumann, 2012, Ch. 6.1.2). It's constructed to demarcate systems with gravitational radiation from those without. According to its inventor himself, "nobody has fully understood" it (in: Kennefick, 2007, p. 208).

Another line of enquiry should examine the role GW and gravitational energy play in relativistic astrophysics. As I argued in §3.2, the inference from failure of energy conservation to energy transfer – i.e. the ascription of energy to GWs or spacetime more generally – is a

[29] More precisely: in the optical limit (see, e.g., Geroch, 2013, Ch. 13).

non-sequitur. One may expect further insights from studying other astrophysical processes, especially those during which energy is extracted from a gravitating source, such as the energy gain of particle passing through the ergosphere of a Kerr Black Hole.

This chapter:

We saw that the standard arguments on which our *expectation* rests that gravitational waves – representative of general-relativistic gravity, more generally – carry energy are problematic. They presuppose assumptions one may well question.

The next chapter:

Let's move beyond our intuitions and hunches, shaped – for better or worse – by gravitational waves. What are specific candidates for gravitational energy-stress in General Relativity? Do they settle the matter more satisfactorily?

III. Local Gravitational Energy and Energy Conservation in General Relativity

Abstract:

This chapter critically examines energy-momentum conservation and local (differential) notions of gravitational energy in General Relativity (GR). On the one hand, I argue that energy-momentum of matter is indeed locally (differentially) conserved: physical matter energy-momentum 4-currents possess no genuine sinks/sources. On the other hand, global (integral) energy-momentum conservation is contingent on spacetime symmetries. Local gravitational energy-momentum is found to be a supererogatory notion. Various explicit proposals for local gravitational energy-momentum are investigated and found wanting. Besides pseudotensors, the proposals considered include those of Lorentz and Levi-Civita, Pitts and Baker. It is concluded that the ontological commitment we ought to have towards gravitational energy in GR mimics the natural anti-realism/eliminativism towards apparent forces in Newtonian Mechanics.

Key words: Gravitational energy, energy conservation, General Relativity, inertial frames, pseudotensors, cosmological constant, energy-momentum tensor

III.1. Introduction

Energy and its conservation are a pivotal part of almost all of physics. From early on, attempts to define energy for the gravitational field in GR sparked controversy. Progress in this regard was in part responsible for GR's reinvigoration as mainstream physics from the 1950s on (see Schutz, 2012; Kennefick, 2007, Ch. 11, 12). But the quest for a fully satisfactory account of gravitational energy continues. In the following, I examine whether in GR gravitational energy – the energy ascribable to spacetime itself – is a meaningful local (differential) notion: does there exist something like gravitational energy-momentum density? A related question concerns the validity of energy-momentum conservation: does *non-gravitational*/*matter* energy-momentum 4-currents possess sources or sinks?

The aim of this chapter is conceptual analysis: can or should one endorse realism about local gravitational energy in GR, drawing only on the latter's fundamental concepts? (I will steer clear of the question of whether a *higher-level* concept of gravitational energy exacts some form of realist commitment – whether, for instance, an effective gravitational energy, definable in a certain domain, counts as a "real pattern" in the sense of Dennett, 1991.[30]) My objective is conceptual clarification: what can be said about local gravitational energy within GR's fundamental ontology and ideology?

I contest the existence of local gravitational energy in GR. It will be argued to be an eliminable concept, not meriting a realist stance. Nonetheless, there is considerable continuity between GR and its precursors. Locally, the energy-momentum of matter is indeed conserved, with no need for gravitational energy contributions to restore an energy balance. The difference between GR and its precursors relevant here lies solely in the fact that GR's inertial frames are only defined locally, in contrast to the globally defined ones in Classical Mechanics (CM) or Special Relativity (SR).

Those views are widespread amongst relativists. This chapter will seek to vindicate them. To date, a systematic review and evaluation of the arguments in favour of them, as well as an exposition of a coherent account are pending. This I will attempt to provide.

[30] Such an attempt is found in Read, 2017, to whom I respond in Ch. IV.

I will proceed as follows. In **§2**, I will first (**§2.1**) explore local energy-momentum conservation in generic spacetimes: Can the vanishing covariant derivative of the energy-momentum tensor be interpreted as a local energy conservation law? What are the role and status of gravitational energy-momentum in such conservation laws? **§2.2** zooms in on symmetric spacetimes. In particular, I address the question of global energy conservation. **§3** is devoted to local representations of *gravitational* energy. I first (**§3.1**) criticise Lorentz and Levi-Civita's tensorial proposal and elaborate on the necessity of non-tensorial expressions for gravitational energy-momentum. As an example, I subsequently (**§3.2**) study pseudotensorial approaches via the Noether Theorems, and expound their main problems. **§3.3** discusses Pitts' proposal for an infinitely many component object for gravitational energy. Another proposal, based on the cosmological constant, is studied in **§3.4**. A summary of my conclusions is provided in **§4**.

I build upon pioneering work by Hoefer (2000). He characterises what he declares the "received view" of local representations of gravitational energy-momentum by the following three claims. First, one postulates the vanishing of the covariant divergence of the energy-momentum tensor of matter, $\nabla_b T^{ab} = 0$. Secondly, since in general it does not satisfy a proper continuity equation, $\partial_b T^{ab} \not\equiv 0$, the vanishing covariant divergence of the energy-momentum tensor forms a conservation law proper only for the sum of material *plus* gravitational energy-momentum. That is: One posits contributions from gravitational energy, not included in T^{ab}. Neglecting these contributions is supposed to result in *apparent* non-conservation of energy-momentum, which is what $\partial_b T^{ab} \not\equiv 0$ is interpreted as. Thirdly, such gravitational energy contributions are then lumped into one object, the so-called "pseudo-tensor" t^{ab}, inferred only indirectly, such that a continuity equation of the type $\partial_b(T^{ab} + t^{ab}) = 0$ is restored.

Hoefer rejects this received view on two grounds. First, the non-uniqueness of t^{ab}, in his opinion, undermines its well-definedness. Secondly, he takes its non-tensorial nature to obviate interpreting t^{ab} as an intrinsically meaningful, well-defined quantity. With the jury still out on future progress with respect to quasi-local definitions of energy in GR (which try to associate energy-momentum (density) not with individual spacetime points, but only with extended, finite regions of spacetime), Hoefer enjoins us to relinquish both the notion of local gravitational energy-momentum and conservation of energy-momentum in GR altogether.

Hoefer's arguments are not likely to sway believers in gravitational energy. To begin with, his claim that the gravitational energy-momentum pseudotensor t^{ab} is inferred "only indirectly", insinuating its *ad-hoc* character, is misleading: as we will sketch in §3.3, t^{ab} arises in a *direct* way no less naturally than energy-momentum in other field theories.

Hoefer's objections, too, call for further clarification. For one, the nature of the ambiguity and non-uniqueness of pseudotensors must be fleshed out: what does it consist in? How severe is it? Is it physically significant? Furthermore, vis-à-vis Hoefer's objection to the lack of coordinate invariance, one may be tempted to bite the bullet: what is inherently wrong with non-tensors? In itself, an object's non-tensoriality need not undercut its meaningfulness: the connection coefficients, $\Gamma^a_{bc} = \frac{1}{2} g^{ad}(\partial_c g_{db} + \partial_b g_{dc} - \partial_d g_{bc})$, attest to that. They are endowed with both a bona fide physical and geometric meaning, representing inertial structure, and (in the language of fiber bundles) connecting the fibers of the vector bundle over different points of the base manifold, respectively. Even if one shares Hoefer's scepticism towards pseudotensors, are they indeed the *only* way to locally represent gravitational energy? Might there exist other approaches? The answer is yes. Bel and Robinson, for instance, have proposed a tensor which mimics the way the electromagnetic energy-momentum tensor is constructed from the Faraday tensor (see, for instance Horský & Novotný, 1969, sect.III.4). To be sure, as a candidate for gravitational energy the Bel-Robinson tensor is not immune to criticism: neither it nor any of its powers possess the right units of energy-momentum. It would thus mandate a novel constant of nature, i.e. an additional structure, absent in GR *simpliciter*.

This leads us to Lam's recent refinement of Hoefer's critique (Lam, 2011). He adverts to the need to introduce additional structure, in order for local energy-momentum conservation to hold, and for gravitational energy-momentum to be well-defined.

Lam discusses the important special case of spacetimes instantiating a so-called Killing field (see §2.2). He argues that only such spacetimes allow $\nabla_b T^{ab} = 0$ to be interpreted as a well-defined local (and global) notion of energy-momentum conservation.

Lam suggests the following construal: "[…] a time-like Killing vector field can be understood as defining a global inertial frame, which can represent a global family of inertial observers all at rest with each other" (p. 5). This global inertial frame, Lam asserts, "[…] can be understood in a certain sense as a nondynamical background structure with respect to which integral nongravitational energy-momentum can be obtained. […] A fully dynamical metric field would prevent the existence of such global symmetries. […] In this sense, the nature of gravitational energy seems to be linked to the failure of certain global symmetries and, most importantly to the lack [of] non-dynamical background structures, that is to background independence" (ibid). In other words: GR's background independence (absence of nondynamical objects), Lam argues, subverts energy-momentum.

Lam deserves credit for honing Hoefer's analysis of the pseudotensor by specifying what in his opinion their ambiguity consists in: the freedom to insert a term of the form $\partial_c U^{a[bc]}$, i.e. antisymmetric in *b* and *c*, into the continuity equation without altering it: $\partial_b\left(T^{ab} + t^{ab} + \partial_c U^{a[bc]}\right) \equiv \partial_b(T^{ab} + t^{ab})$. The pseudotensor, Lam concludes, hence lacks uniqueness in that any $t^{ab} = \partial_c U^{a[bc]} - \frac{1}{2\kappa} G^{ab}$ equally satisfies the continuity equation. (Here we have exploited the Einstein Equations, $G^{ab} = 2\kappa T^{ab}$.)

He also sharpens Hoefer's principal argument against pseudotensors, their coordinate-dependence. "In particular, at any spacetime point or along any world-line, there is a coordinate system in which $[t^{ab}]$ vanishes. Since different coordinate representations are just different mathematical descriptions, relevant physical entities are usually taken to correspond to coordinate-independent entities […]. So, the coordinate […] dependence of $[t^{ab}]$ shows that there is no (unique) gravitational energy-momentum, in the sense that such a quantity cannot be in general unambiguously defined at any spacetime point" (p. 6).

Notwithstanding its improvement over Hoefer's account, Lam's own remains unsatisfactory. For one, as we will see below, while it is true that the existence of Killing symmetries is sufficient for the validity of local energy-momentum conservation, it is not necessary – contrary to what Lam seems to suggest. More precisely, Lam implicitly presupposes a gratuitously strong definition of *inertial* reference frames for GR – one that posits the existence of Killing fields.[31]

[31] Presumably, Lam has in mind something like Earman & Friedman's definition of inertial frames explicitly determined by a time-like *Killing* vector (1973, pp. 353). It remains unclear, however, why a natural general-relativistic generalisation of the notion of inertial frame in Newtonian Gravity (e.g. p. 332), should employ such a strong prerequisite. For one, strictly speaking this restricts the existence of inertial frames *a priori* to stationary spacetimes. Thereby, one precludes any remotely realistic scenarios.

Lam's elucidations regarding the problematic status of, and link between, energy-momentum conservation and gravitational energy are deficient, as well. The connection between energy-momentum conservation, gravitational energy, symmetries and GR's background independence remains hand-waving – not least, since a precise definition of background independence is notoriously elusive (see, e.g. Pooley, 2015, esp. sect. 7; Read, unpublished).

As in Hoefer's case, a proponent of gravitational energy is likely to counter Lam's ambiguity objection to pseudotensors by asking in what ways the situation differs from other field theories. It is crucial to note that the way pseudotensors arise in GR is in a completely *standard* way via the Noether Theorems. Lam mentions them in a footnote. But he does not expand on the connection. This would have pre-empted a potential misunderstanding: the form of the continuity equation that both Hoefer and Lam discuss is restricted to special (unimodular) coordinates, satisfying $|g| := \det(g_{\mu\nu}) = 1$. But such a restriction is unnecessary. More importantly, it is *not* the source of the (vicious) coordinate-dependence of pseudotensorial expressions.

Furthermore, a naïve reading of what Lam offers as another way to understand the pseudotensor – namely as nonlinear correction to the linearised Einstein tensor, i.e. higher perturbative orders[32] – is problematic. It's predicated on the premise that the Einstein Equations' nonlinearity reflects the fact that gravitational energy acts as a source of the gravitational field itself. Two examples demonstrate that this can't be quite right. First, a (consistent) implementation of self-energy in standard Newtonian Gravity leads to a non-linear theory, too. Yet, its gravitational energy-stress tensor remains well-defined, even tensorial (Giulini, 1997). (An example of a non-gravitational theory that, despite its ferocious nonlinearity, admits of a fairly unproblematic notion of energy and its conservation is given by the Navier-Stokes Equations.) Secondly, the desire to *explain* GR's nonlinearity stems from

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The motivation behind Earman and Friedman's stipulation is to preclude spacetimes, such as Gödel's, in which no globally defined reference frames exist. These they dismiss for not allowing of a "globally consistent time sense"(p. 353).
Suffice it to say here that Earman and Friedman fall short of a cogent argument why the non-orientability of certain spacetimes is supposed to be linked to inertial frames specifically: why assume that the problems of how to interpret such spacetimes originate in the non-existence of global inertial frames? It seems no less plausible to stipulate a definition of inertial frames that applies to such non-orientable spacetimes – and dismiss the latter (should one feel so inclined, at all) as unphysical on *independent* grounds.
By contradistinction, I'll adopt the weaker and more natural notion of general-relativistic inertial frames, identifying them with freely falling frames (see e.g. DiSalle, 2009, sect. 2.9).
[32] E.g. Hobson et. al., 2006, p. 473 for a similar claim.

comparing it to *linear* theories. Both physically – with the familiar linear theories being non-fundamental, and structurally – such a comparison is implausible, though: rather, one should compare GR with likewise (fairly) fundamental, non-linear vectorial Yang-Mills-type theories (cf. Deser, 1970). Again, the immediate lesson is: nonlinearity *isn't* the source of GR's trouble with its gravitational field-energy.

Similarly, Lam's objection of coordinate-dependence is not persuasive. Coordinate-dependence by itself need not be baneful: as long as a coordinate-dependent structure has the same symmetry group as the one of the *given* spacetime, the coordinate dependence is benign. It becomes vicious only if the coordinate-dependent object is not invariant under the spacetime's symmetry transformations. I revert to this in §3.3. Although I concur with Lam's thought that coordinate-dependence of the type pseudotensors elicit amounts to something akin to an unphysical gauge-fixing, we must clarify whether pseudotensors are benign or vicious in the sense just mentioned.

Hoefer and Lam raise deep questions, and I am largely in agreement with their positions. This chapter intends to supplement their work, attempting to fill some of the indicated lacunae and to provide a systematic account of local energy-momentum conservation and local notions of gravitational energy within GR.

III.2. Energy-momentum conservation in local form

III.2.1. Energy-momentum conservation in generic spacetimes

In lieu of the ordinary (partial) zero-divergence for the matter energy-momentum tensor, in GR we have the zero *covariant* divergence, $\nabla_b T^{ab} = 0$. Here, $T_{ab} = -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}\mathcal{L}\right)$ denotes the matter energy-momentum tensor, with the matter Lagrangian $\mathcal{L} = \mathcal{L}\left(\mathrm{g}_{ab}; \psi, \nabla_a\psi, \nabla_{a,b}\psi, ..\right)$ for the (tensorially generic[33]) matter fields ψ.[34] Throughout, I'll

[33] For how spinorial matter fits into the picture see Pitts (2011).

[34] Despite representing gravity via variational derivatives of a gravitational Lagrangian, Einstein never systematically defined energy-momentum tensor variationally nor did he work with matter Lagrangians (see Pitts, 2016a).

assume a Lagrangian approach also for GR's matter sector. The vanishing of the covariant divergence holds independently of the Einstein Equations: it follows from $\mathrm{diff}(\mathcal{M})$-invariance, the imposition that the dynamical matter variables ψ satisfy the Euler-Lagrange Equations, and the above form of the matter Lagrangian (see e.g. Hobson et al., 2006, Ch. 19.12 for technical details).

In the literature (e.g. Bergmann, 1976, pp. 193; Brown & Brading, 2002; Padmanabhan, 2010, p. 213), it is sometimes stated that $\nabla_b T^{ab} = 0$ does not express local energy-momentum conservation simpliciter. Rather, it is said to denote the degree to which conservation is *violated*:

$$\partial_b T^{ab} = -2\Gamma^{(a}_{bc}T^{b)c}(\not\equiv 0) \quad (1).$$

quantifies the extent to which energy-stress density/flux is no longer source-free/sink-free in *generic* reference frames.

This is perhaps not the most perspicuous way of formulating the problem. It distracts from what is *special* about GR's energy-momentum: in which regard does the above situation differ from conservation of the external electric 4-current, $\nabla_a j^a_{(ext)} = 0$? After all, it *too* can be rewritten to yield apparent "sources/sinks" in generic reference frames:

$$\partial_a j^a_{(ext)} = -\Gamma^a_{ab} j^b_{(ext)}(\not\equiv 0) \quad (2).$$

Only for unimodular coordinates (modulo global re-scalings), i.e. coordinates satisfying $\sqrt{|g|} = const.$, does the "source term" of the continuity equation on the r.h.s. vanish.[35]

This is arguably rooted in his scepticism towards any (classical) matter theories. For him, any classical matter theory was merely preliminary and phenomenological – to be superseded by a future quantum treatment. By contrast, he had considerable trust in the fundamental correctness of GR's account of (pure) gravity (see e.g. Lehmkuhl, 2017, ms for details).

[35] Note that with this fixed volume element, unimodularity introduces an absolute structure, extraneous and inimical to GR's overall non-absoluteness (cf., for instance, Anderson, 1967).
The choice of unimodular coordinates is directly related to inertial frames – a concept that will occupy centre stage in the subsequent analysis. With respect to their global/integral properties, unimodular coordinates can be regarded as the closest counterparts of Lorentz/Euclidean coordinates on generically curved manifolds: they preserve a constant volume element.
$\partial_a j^a_{(ext)} = -\Gamma^a_{ab} j^b_{(ext)}$ yields a continuity equation proper, only if one selects as reference frames those counterparts of *inertial reference frames in Minkowski spacetime*. But the latter cease to be distinguished in *GR*. One conclusion of my subsequent arguments is that we should take *GR's* notion of inertial structure more seriously: we need not (and should not) import it from non-GR theories.

In that case, however, the restriction to unimodular coordinates can be lifted. By considering the vector density of weight-1 (see e.g. Schrödinger, 1950, Ch. II for more on tensor densities) $\tilde{j}^a_{(ext)} \coloneqq \sqrt{|g|} j^a_{(ext)}$, one obtains a 4-current that satisfies a continuity equation in *every* coordinate system:

$$\nabla_a \tilde{j}^a_{(ext)} \equiv \partial_a \tilde{j}^a_{(ext)} \equiv 0 \quad (3).$$

The electric charge flux (weight-1 density) $\tilde{j}^a_{(ext)}$ thus defined is locally conserved without qualification. (Recall that $\sqrt{|g}$ gives an infinitesimal volume element. Hence, the interpretation of (3) is conservation of electric charge, contained within an infinitesimal volume.) Is a similar move – the transition from a tensor to a tensor *density* (of whatever weight) – available also in the case of $\nabla_b T^{ab} = 0$, whereby we could restore our intuitive sense of conservation, captured in the ordinary 4-divergence of a 4-current?

To see what is troublesome about energy-momentum in GR specifically, we must be more circumspect. Consider an arbitrary time-like vector ξ. Along its direction, one can define the energy-momentum 4-current $j^a[\xi] \coloneqq T^a_b \xi^b$. Unless ξ is special (in a sense to be made precise presently), the covariant divergence of this 4-current does *not* vanish, $\nabla_a j^a[\xi] = T_{ab} \nabla^a \xi^b \neq 0$. This yields an ordinary continuity equation with *non-vanishing* source/sink terms in any reference frame: in *no* reference frame other than the special ones (in which $\nabla^a \xi^b$=0) is $j^a[\xi]$ locally conserved. (The transition to a vector density is of no avail here.)

In short: The external electric 4-current satisfies an ordinary continuity equation for *some* coordinate systems – namely unimodular ones. (The corresponding weight-1 density does so even for *every* coordinate system.) By contrast, whether the energy-momentum 4-current $j^a[\xi]$ satisfies an ordinary continuity equation (for some reference frames) depends on the choice of its direction ξ.

In generic ("non-symmetric", see §2.2) spacetimes, what are these distinguished directions of energy-momentum flux along which one would register the absence of sinks/sources? For inspiration, consider a free-falling observer ξ. In her proper (comoving) reference frame γ, we have $\xi^a|_\gamma = \delta^a_0$ and $\Gamma^a_{bc}|_\gamma = 0$. (Here, we'll use the coordinates adapted to this frame, i.e. inertial ones along γ.) The corresponding energy-momentum 4-current $j^a[\xi]$ that ξ measures *along her worldline* is source-/sinkfree: $\partial_a j^a[\xi]|_\gamma = 0$. Sources/sinks would appear for ξ only, when adopting reference frames other than her proper one. The energy-momentum 4-current along the worldline of observers *not* in free-fall, i.e. non-inertial observers, is locally conserved in *no* reference frame.

How to construe these sinks/sources in local energy-momentum 4-currents for generic reference frames and along generic directions? Does it *seriously* jeopardise *real* energy-momentum conservation (in a sense to be made precise)? In particular, if energy-momentum isn't conserved, is it because we have neglected gravitational contributions to it – having considered only energy-momentum of (non-gravitational) matter? For an answer, I will first identify the class of ξs picked out by the apparent anisotropy of local energy-momentum conservation, as well as the reference frames for which energy-momentum is un-controversially conserved locally.

A prior side-glance to Classical Mechanics (CM) is instructive. There, an analogous problem arises for apparent/fictitious/inertial forces, e.g. the Coriolis or centrifugal force. They flout Newton's Third Law of action-reaction. In contrast to *genuine* ones, apparent forces do not mediate physical interactions. They are not causes nor are they caused (cf. Nerlich, 1989, sect. 5). We do not ascribe them the status of entities "out there", forces as real as, say, the Lorentz force. They are more like shadows: existentially parasitic, causally inert and explanatorily non-fundamental. We are wont to conceive of them as springing from descriptions in non-inertial coordinate systems. Ontologically, apparent forces are reduced to inertial motion, as it *appears* from non-inertial reference frames. They are artefacts of physically artificial descriptions (e.g. Maudlin, 2012, pp. 23 fn. 7).

Inertial reference frames are inherently distinguished by "natural" (as opposed to "forced", or caused, motion (cf., for instance, Brown 2007, p. 163). This class of kinematic states, privileged as default motion in which every body persists unless external forces act on it, is furnished by the theory's inertial structure. Bodies moving inertially do *not* call for deeper explanations of this kind of motion (Nerlich, 1979; Janssen, 2009). Only non-inertial motion does: when a body *deviates* from inertial motion we ask for causes[36] – in the form of external forces.

[36] Nothing here hinges on any metaphysically thick understanding of causation. The above should be acceptable even to those sceptical of causation (such as Norton, 2003, 2009). The thrust of the above point is primarily explanatory: causes, in the intended broad construal, are the deeper explanantia called for, in order to account for certain phenomena.

"Inertially framed" accounts afford simpler explanations than a "non-inertially framed" one. They are adapted to the spacetime geometry, thereby making explicit what merits realist commitment. Conversely, extra terms that arise in non-inertial frames are representational (or perspectival) artefacts of physically unnatural descriptions.

How do these remarks bear upon GR's local energy-momentum conservation? First, they answer which directions *are* distinguished in generic spacetimes: directions along natural/inertial motion. In GR, this is motion along (time-like) geodesics (for systematic and historical details, see e.g. DiSalle, 2009, esp.2.9; Petkov, 2012; Knox, 2013, esp. sect. 2). Secondly, by the same token, we also get an answer to what the privileged reference frames are: ditto, the inertial ones. In GR, the latter are identified as free-fall frames γ, the coordinates adjusted to them being normal coordinates. (Henceforth, I will use Fermi's.) By construction, their connection coefficients vanish, $\Gamma^{a}_{bc}|_{\gamma} = 0$.

For a geodesic/free-fall trajectory ζ, the inertial frame γ is comoving. Concomitantly, in the adapted inertial coordinates, $\zeta^{a} = const$. In consequence, the energy-momentum 4-current along free-fall trajectories ζ is locally conserved, $\partial_{a} j^{a}[\zeta]|_{\gamma} = 0$.

The lesson to be drawn is this: *apparent* violations do not evince a *real* break-down of local energy conservation. Nor do they signal that we have neglected some (presumably: gravitational) energy contributions. Rather, such "violations" are artefacts of an *unphysical* direction for the 4-current or of adopting non-inertial frames.[37] Neither should unsettle us. GR's matter energy-momentum 4-current is free of sinks/sources *no less than* in CM or SR. In these theories, as well, energy can *appear* not to be locally conserved, when adopting non-inertial/accelerated reference frames.

GR and pre-GR theories differ, of course, in their specific inertial structure, i.e. what they posit as privileged "natural motion". I will outline the ramifications that these differences entail for

[37] Landau & Lifshitz (1975, p. 283) insist (with neither argumentation nor even explication) that any serious candidate for energy must satisfy a continuity equations in *all* reference frames. I reject this assumption. There is nothing wrong with (and hence nothing to ameliorate in) an equation that does *not* take its simplest form in non-inertial frames.
Presumably, what motivates Landau and Lifshitz's reasoning is Einstein's (erroneous) interpretation of general covariance as an extension of the Relativity Principle, asserting the equivalence of *all* reference frames (cf. Norton, 1985).

global energy-momentum conservation shortly (§2.2). But first, it is apposite to discuss GR's local energy-momentum conservation from a different angle, linking it to gravitational energy.

Following Einstein's 1916 GR review paper (see Hoefer, 2000, p. 191), $\partial_b T^{ab} = -2\Gamma^{(a}_{bc}T^{b)c}$ has occasionally (e.g. Brading & Brown, 2002, p. 17) been dubbed the "response equation". Putatively, it captures the exchange of energy-momentum between gravitational and ordinary energy-momentum – gravity's back-reaction upon matter (e.g. Weinberg, 1972, p.166; Hobson et al, 2006, p. 181 or Rindler, 2009, p. 299). I commented already on how focusing of the decomposition of T^{ab}'s vanishing covariant divergence, $0 \equiv \nabla_b T^{ab} = \partial_b T^{ab} + 2\Gamma^{(a}_{bc}T^{b)c}$, is misleading.[38]

One can reformulate the response equation interpretation (REI) more judiciously.[39] Recall that for generic spacetimes and ξ, the covariant divergence of the energy-momentum flux along ξ does not vanish:

$$\nabla_a j^a[\xi] = T^a_b \nabla_a \xi^b (\not\equiv 0) \quad (4).$$

(Equivalently, consider the weight-1 density $\tilde{j}^a[\xi] := \sqrt{|g|}j^a[\xi]$. The corresponding continuity equation with source terms is: $\nabla_a \tilde{j}^a[\xi] \equiv \partial_a \tilde{j}^a[\xi] = \sqrt{|g|}T^a_b \nabla_a \xi^b$.) According to the ameliorated REI, the non-vanishing $\nabla_a j^a[\xi]$ reflects the intertwinement of gravitational and matter energy-momentum. The presence of sinks/sources in the 4-current on the r.h.s. is attributed to the *neglect* of gravitational energy contributions.

What is the relationship then between the presence of gravitational energy and the presence of sinks/sources? Is the former *necessary* for the latter? That is: Is gravitational energy the (only) reason for the failure of the conservation of matter (non-gravitational) energy-momentum? This seems implausible. In inertial frames, the r.h.s. of $\nabla_a j^a[\xi] = T^a_b \nabla_a \xi^b$ reduces to $T^a_b \partial_a \xi^b$. (NB: ξ^b still remains an arbitrary vector field.) The latter, however, is not straightforwardly related to gravity: one would expect gravitational degrees of freedom to be encoded in metric-dependent quantities – not an arbitrary vector field. The fact that the 4-current $j^a[\xi]$ contains sinks/sources thus would *not* be related to gravity, either. (One may object to evaluating $\nabla_a j^a[\xi]$ in inertial reference frames, as in the latter, gravity has been

[38] Due to his own interpretation of GR, Einstein had no such qualms talking about "inertial" and "gravitational" components of the decomposition of the geodesic equation (see Lehmkuhl 2010).
[39] The improved form of the REI avoids the objections against its usual form (cf. Read, 2017).

"geometrised away". Rather than an objection, this objection just anticipates the conclusion *for* which I shall ultimately argue.)

The REI may be understood as taking the presence of gravitational energy to be *sufficient* for the presence of sinks/sources in $j^a[\xi]$. Equivalently, by contraposition, $\nabla_a j^a[\xi] = 0$ should imply the *absence* of gravitational energy. Prima facie, this makes sense: if the matter energy-momentum 4-current contains no sources/sinks, gravitational energy does not contribute to the energy balance.

Does anything more interesting follow from this claim of the REI for gravitational energy? For an answer, first recall our earlier discussion that in generic spacetimes energy-momentum 4-currents along directions other than along inertial trajectories are physically ungrounded: we need not extend our realist commitments to such quantities. Now consider a situation (in a non-symmetric spacetime) with gravitational energy present. According to the REI (again by contraposition), it would follow that $\nabla_a j^a[\xi] \neq 0$. This is possible (in non-symmetric spacetimes) only for ξs that describe *non-inertial* trajectories or a description in non-inertial reference frames. Hence, this non-conserved form of $j^a[\xi]$ is barred from our realist commitment. In short: The REI implies that gravitational energy leads to what would *appear* as a violation of local energy-momentum conservation. Our analysis of inertial motion, however, disclosed that the energy-momentum 4-currents that apparently are not locally conserved are *unphysical*.

Consequently, according to the REI – as a plausible link between gravitational energy and local energy-momentum conservation – gravitational energy is an idle wheel: *real* energy-momentum – energy-momentum meriting realist commitment – is locally conserved in GR; gravity does *not* contribute to the energy balance equation.

This renders precise and corroborates Norton's conjecture that GR, as a theory that "geometrises away"[40] gravity, also compromises gravitational energy. Like apparent forces in

[40] "*Geometrising* away" must not be confused with "*transformed* away". The former denotes the fact that gravitational phenomena are not attributed to external forces which deflect particles from their rectilinear inertial paths, (see, for instance, Maudlin, 2012, Ch. 6). Rather, they are reconceptualised as manifestations of a non-Newtonian/non-Minkowskian inertial structure. "Transforming away", on the other hand, suggests that one could make these effects disappear though a suitable choice of coordinates.
This is not the case for GR: Consider, for instance, the wave-equation for the Faraday tensor in general-relativistic Einstein-Maxwell Theory with external current J^a. It contains gravity-related curvature terms (see Read et al., 2017 (ms), sect. 2,3 for details):

$$\nabla_c \nabla^c F_{ab} = F^d_{\ [b} R_{a]d} - R_{abcd} F^{cd} - \nabla_{[b} J_{a]}.$$

CM, gravity is not a force arising from gravitational interaction, a *real* entity. In GR, gravity is not a force: gravitational phenomena are manifestations of non-Minkowskian inertial structure (see Earman & Friedman, 1973, sect. 5; Norton, 2003; Nerlich, 2013).[41] GR thereby inaugurates a shift in what phenomena are in need of explanations in terms external causes (cf. Nerlich, 2013, Ch. 8; Dorato, 2014). Non-vanishing gravitational energy is an artefact of bestowing on directions and non-inertial reference frames a physical significance that in truth (according to GR's standard interpretation) they lack.[42] (Of course, in generic spacetimes – those lacking time-like Killing fields – inertial reference frames exist only "locally": only along a single, privileged (geodesic) time-like path can we find coordinate-systems whose time-like axes move inertially. I return to this later on in §2.2.) In short: Non-vanishing gravitational energy is the result of a spurious realist commitment. I will call this position "eliminativism about gravitational energy".

Taking seriously GR's inertial structure, I argued in this section that local energy-momentum conservation is valid in GR. Apparent violations in GR for certain reference frames and along certain directions merit no more realist commitment than violations of Newton's Third Law by apparent forces. Just as one should be an eliminativist about apparent forces in CM, one should be an eliminativist about those energy-momentum 4-currents in generic GR spacetimes that are not conserved. To special cases, for which energy conservation does hold, we will turn in the next section.

In pre-GR theories, local energy-momentum conservation gives rise to an invariant *global* conservation law, as well: the energy of matter contained in a closed space-like hypersurface remains constant across time. Does this also hold in GR?

III.2.2. Local energy conservation in symmetric spacetimes

In the previous section we considered generic spacetimes, devoid of symmetries. What changes with respect to energy-momentum conservation in a spacetime with symmetries?

As tensors, these curvature-containing terms cannot be eliminated through any choice of coordinates.

[41] Cf. Dewar & Weatherall, 2018, esp. section 5 for the similar case of Newton-Cartan-Theory, a geometrised version of Newtonian Gravity. I'll revert to this in Ch. V.

[42] In Dürr (2018), i.e. Ch. II of this thesis, the case of gravitational waves, and the question whether they carry energy, is discussed. I argue that the standard arguments supposed to demonstrate that they do are not convincing. An alternative account of the spin-up of binary systems is given. Of course, I do not deny the *reality* of gravitational waves and their effects. I only deny that they *transport* energy.

In CM, it is often said (e.g. Landau & Lifshitz, 1976, §6-9) that conservation of energy and linear/angular momentum are correlated with the homogeneity of time and the homogeneity/isotropy of space, respectively, via Noether's First Theorem. (The latter remains neutral, though, about *spacetime* symmetries per se.[43] Rather, it establishes a link between conserved quantities and symmetries of an *action* under rigid coordinate transformations (see Sús, 2017 for a lucid account). But because the relevant coordinates we deal with in CM are spacetime coordinates (rather than coordinates on internal spaces, as they occur in gauge theories), one may identify rigid transformations with time/space translations.)

GR's spacetime symmetries are expressed by means of Killing vectors ξ, the infinitesimal generators of a spacetime's isometries. They are defined via a vanishing Lie-drag of the metric along them

$$0 = \mathfrak{L}_{\xi} g_{ab} = \nabla_{(a}\xi_{b)} \quad (5).$$

Killing vectors give constants of motion along a geodesic $x^a(\tau)$, with the affine parameter τ :

$$\frac{d}{d\tau}(\dot{x}^a \xi_a) = 0 \quad (6).$$

For energy-momentum T^a_b in particular, the existence of a time-/spacelike Killing field ξ gives rise to an energy-momentum 4-current $j^a[\xi] \coloneqq T^a_b\, \xi^b$ that satisfies a local conservation law,

$$\nabla_a j^a[\xi] = T^{ab}\nabla_a \xi_b = -T^{ab}\nabla_b \xi_a = -T^{ab}\nabla_a \xi_b \equiv 0 \quad (7).$$

In inertial frames γ, it even simplifies to a familiar, ordinary continuity equation, $0 \equiv \nabla_a j^a[\xi]|_{\gamma} = \partial_a j^a[\xi]$. (As in §2.1, the transition to the vector density yields a continuity equation in all reference frames: $0 \equiv \nabla_a(\sqrt{|g|}j^a[\xi]) \equiv \partial_a(\sqrt{|g|}j^a[\xi])$.) The Killing field thus provides a distinguished direction along which energy-momentum has no sinks/sources. The status of conservation of an energy-momentum 4- current along a Killing field *now* is the same as that of conservation of the external electric 4-current in Maxwellian electrodynamics.

Assuming that T^a_b has compact support (or benign fall-off conditions), the 4-current $j^a[\xi]$ also gives rise to an associated invariant, *globally* conserved "charge" $Q[\xi] \coloneqq \int_{\Sigma} d\Sigma_a j^a[\xi]$ (with

[43] I am grateful to Brian Pitts (Cambridge) for pressing me on this point.

the directed infinitesimal volume element $d\Sigma_a$): $Q[\xi]$ doesn't depend on the choice of the Cauchy hypersurface Σ (see, for instance, Padmanabhan, 2010, Ch. 6.5 for details).

Spacetimes with Killing symmetries thus admit of both, local and global energy-momentum conservation. Minkowski space, for instance, possesses ten Killing fields, corresponding to the 10-dimensional Lie algebra of the Lorentz group. They are associated with conservation of energy, linear and angular momentum. The coordinates that are adapted to the time-/spacelike Killing vectors correlated with energy-momentum conservation are *globally* defined (as opposed to defined only at a *point* or along a *curve*, as in the GR case for Riemann or Fermi normal coordinates, respectively) inertial coordinates – the familiar Cartesian/Lorentz coordinates.

Contrast the situation with the non-symmetric spacetimes from §2.1. In the absence of Killing symmetries ξ, the 4-current $j^a[\xi]$ is source-/sinkfree merely for the only inherently distinguished directions available in such spacetimes: ξs along *inertial/free-fall* trajectories. The coordinates adapted to the inertial frames are comoving. They are only defined along the inertial paths, not globally. For non-Killing ξ, the 4-current $j^a[\xi]$ does not yield a well-defined global charge: different 3+1-decompositions of spacetime imply different charges, each such slicing in itself being but an arbitrary conventional choice. In particular, the charges are not conserved across time: for a 3+1-decomposition of the manifold into the one-parameter family of spacelike hypersurfaces $\{\Sigma_\sigma : \sigma\}$, we have:

$$\frac{d}{d\sigma}\int_{\Sigma_\sigma} d\Sigma_a j^a[\xi] \not\equiv 0 \quad (8).$$

Generic spacetimes lack Killing symmetries. In particular, in generic spacetimes energy-momentum fails to be conserved *globally*. This is illustrated by the "singularly striking example" (Schrödinger, 1950, p. 105) of the decrease of energy in a closed bounded universe (cf. Misner et al., 1974, §19.4 for technical details): "In simple models the loss [of energy contained in a closed 3-volume] can be computed and equals the amount of work the pressure would have to do to increase the volume, if a piston had to be pushed back as in the case of an adiabatically expanding volume of gas." However, as Schrödinger stresses, this is a *fictional* account: there is no piston nor any boundary through which energy could escape. Global energy conservation just ceases to be valid for the expanding universe.

Strictly speaking, such inertial frames, in which energy-momentum is conserved, have zero-volume. Consequently, matter energy-momentum is conserved only in systems of zero-volume: prima facie, energy-momentum would thus fail to be conserved in *any* systems of interest. Only point-/line-/plane-/hyperplane-like things could happen in inertial frames, precluding nearly everything physicists like to study. Is this an inacceptable consequence, tantamount to a reductio?

Two considerations attenuate the objection. First, many systems of interest, such as the chemical energy in my car's engine, are *sufficiently* small: relative to the relevant scales, they indeed occupy zero-volume. For all practical purposes these systems' energy-momenta then are conserved. (From a *Newtonian* perspective, their gravitational potential energy does not change.) Secondly, also cases of non-negligible energy-momentum non-conservation can be handled *for most practical purposes* – within a certain regime. Notice that the degree of non-conservation is well-quantifiable. Within some degree of accuracy, and within suitably small world-tubes around inertial paths[44], it is thus possible to restore the *apparent* conservation. One only needs to add the "missing" bits by fiat: From a *Newtonian* perspective, these would correspond to the Newtonian gravitational potential energy (and its post-Newtonian correction) to the non-gravitational energy-balance. But from the more fundamental perspective of *GR*, this potential gravitational energy is *fictitious*: They are "translations" (or projections) of *GR* phenomena into a *pre-GR* framework.[45] (This fictional "Newtonised" account ceases to be available beyond a certain degree of accuracy, and in particular, if the curvature effects are very strong even for the relevant volume scales.)

This brings us back to GR's gravitational energy. In §2.1-2, we achieved a transparent account of local energy conservation in GR. In it, gravitational energy was wholly absent. One may take this absence at face value – to the effect that in GR gravitational energy is dispensable. Some will deem this too quick. They may shrug off that absence as irrelevant to the *possibility* of meaningfully defining gravitational energy-momentum. But is there any motivation for that? Carroll (2010), for instance, impugns this. With respect to concocting notions of gravitational

[44] Essentially, this is the case in the regime in which the PPN-formalism is applicable and in which the Equations of Motions (i.e. the field equations for matter/non-gravity) admit of a Lagrangian formulation (cf. Poisson & Will, 2016).

[45] In [Dürr, 2018a], i.e. Ch. II of this thesis, I clarify in the context of gravitational waves the non-trivial difference between violation of energy-momentum and energy momentum depletion via *transport* (cf. also Weatherall, 2016, Ch.2, fn 103).

energy to restore the violation of global energy conservation, he writes: “the entire point of this exercise is to explain what’s going on in GR to people who aren’t familiar with the mathematical details”. Yet, it seems fair to countenance that our analysis of energy-momentum conservation may not have been the right starting point for a search of gravitational energy.[46] Next, I will therefore squarely examine specific proposals for local representations of gravitational energy.

III.3. Local gravitational energy

III.3.1. Tensorial hopes?

Only a year after presenting GR in its full form, Einstein applied to it Noether’s Second Theorem (*avant la lettre*). From the invariance under arbitrary coordinate transformations as the general symmetry of GR’s action (modulo surface terms), four continuity equations ensue (see Brading, 2005, for historical and mathematical details):

$$\partial_b \left(\sqrt{|g|}(T_a^b + t_a^{\,b})\right) = 0 \quad (9).$$

Here, $t_a^{\,b}$ is a suitable object, called “energy-momentum pseudotensor”. It corresponds to the canonical Noether-current for the purely gravitational Lagrangian. (More on this shortly.) It is non-unique: due to its anti-symmetry in the upper indices, inserting an arbitrary term of the form $\partial_c \mathfrak{U}_a^{[bc]}$ into the continuity equation leaves the latter unaffected.

The most prominent example of a pseudotensor is Einstein’s:

$$\mathrm{t}_a^b = \frac{1}{\sqrt{|g|}}\left(-\mathfrak{S}\delta_a^b + \left(\frac{\partial \mathfrak{S}}{\partial(\partial_b g_{cd})} - \partial_e \frac{\partial \mathfrak{S}}{\partial\left(\partial_{b,e} g_{cd}\right)}\right)\partial_a g_{cd}\right) \quad (10).$$

Here, $\mathfrak{S} = \sqrt{|g|}g^{ab}\Gamma^d_{a[b}\Gamma^c_{c]d}$ is the so-called the truncated/“$\Gamma\Gamma$-”Lagrangian.

Four features of the Einstein pseudotensor stand out, underscoring the continuity with other field theories. Firstly, like other energy-momenta from relativistic field theory, it contains solely first derivatives of the field variables, g_{ab}. Einstein’s pseudotensor is constructed fully analogously to energy-momentum in other field theories via the customary Noetherian

[46] Suppose one shares this view. That is: Suppose that one disputes that considerations of energy conservation have a direct bearing on gravitational energy. Then, the simple Geroch-Malament argument against a local gravitational energy in GR that Dewar & Weatherall (2018, pp. 1) cite is blocked.

machinery.[47] The reason is that rather than the full Einstein-Hilbert Lagrangian (plus nondynamical and boundary terms) we may utilise the $\Gamma\Gamma$-Lagrangian, itself containing only first derivatives of g_{ab} (see e.g. Hobson et al., 2006, Ch. 19; Poisson, 2004, Ch. 4.1 for technical details). Secondly, Einstein's pseudotensor is index-asymmetric. This mars its utility for defining angular momentum. But the shortcoming can be amended by the Belinfante-Rosenfeld symmetrisation. This technique is familiar from the likewise non-symmetric energy-momenta in hydro- or electrodynamics. (The technicalities shall not detain us.) Thirdly, although the Einstein pseudotensor transforms tensorially only under *affine* transformations (in particular linear coordinate transformations), the continuity equation, $\partial_b\left(\sqrt{|g|}(T_a^b + \mathfrak{t}_a^b)\right) = 0$, is valid for *every* coordinate system. Fourthly, the weak-field limit reproduces the classical potential energy, and yields reasonable "kinetic" terms for gravitational waves (see e.g. Maggiore, 2007, Ch. 1-3).

Yet, GR's general covariance makes things a little more delicate, when it comes to the symmetrisation procedure and the non-tensoriality. Leclerc (2006, p. 3) cautions that the Belinfante-Rosenfeld symmetrisation presupposes a distinction of certain coordinates inherently not warranted in GR: "The Belinfante procedure relies on the Noether current corresponding to global Poincaré (coordinate) transformations. Certainly, any diffeomorphism invariant action will also be globally Poincaré invariant, but there is no apparent need, a priori, to favor a certain subgroup. In our opinion, this is against the spirit of general relativity. (For instance, in [GR] with cosmological constant, the de Sitter subgroup is at least equally well justified.)" So, if one requires that a suitable candidate for gravitational energy-momentum be index-symmetric, and given that the Belinfante-Rosenfeld symmetrisation exalts certain symmetries *in an ad-hoc way*, then Einstein's pseudotensor seems just not suitable. One might counter: don't free-fall inertial frames already privilege the Poincaré transformation? Consequently, the Poincaré group would seem already distinguished. However, it is not obvious in which way that fact is relevant: in inertial (i.e. freely-falling) frames the Einstein pseudotensor vanishes, after all. But even if one accepts the argument, it does not address a crucial point: all inertial frames are related via *local* (point-dependent) Poincaré transformations relate only inertial frames – not only global (point-independent/rigid) ones.

[47] See Schrödinger, 1950, Ch.XI; Dirac, 1975, Ch. 31, 31 for a more convenient expression.

The other feature of pseudotensors – their non-tensorial nature – looks even more suspect in light of GR's geometric, coordinate-free spirit. Doesn't non-tensoriality conflict with the invariance one would naturally demand of real objects? Before pursuing this further in §3.2, we should enquire into the necessity of pseudotensors: could pseudotensors for local representations of gravitational energy-momentum perhaps be avoided? Several authoritative texts (e.g. Misner et al., 1974, p. 467), deny this, pointing to the Equivalence Principle. Since gravity, the argument goes, can always be made to vanish locally by adopting a free-fall reference frame, gravitational energy can always be "transformed away".

However, the argument has a flaw: it presupposes that the alleged gravitational energy-momentum depends only on *first* derivatives of the metric. Only they could be "transformed away" in suitable coordinates. Why insist on that assumption? Pauli (1981, Ch. 61), for instance, voiced his misgivings along the following lines: gravity manifests itself as curvature (think e.g. of geodesic deviation), represented by the Riemann tensor, $R^d_{abc} = \partial_{[b}\Gamma^d_{c]a} + \Gamma^e_{a[c}\Gamma^d_{b]e}$. But the latter is built from up to *second* derivatives of the metric. Hence, one might expect any natural representation of gravitational energy-momentum likewise to be built from up to second derivatives of the metric.

From the Noetherian perspective on pseudotensorial gravitational energy (more on this in §3.2), such considerations might appear futile: why seek an object with second derivatives, when the terms in the Einstein-Hilbert Lagrangian that contain higher derivatives make no difference to the field equations? In response, note that although a Lagrangian approach is often fertile, it is unclear whether the Lagrangian is more than a mathematical expedient, useful but not physical – comparable to, say, ghost fields in gauge quantum field theory. Suppose that one adopts a merely instrumentalist stance towards the Lagrangian, i.e. regarding the Einstein-Hilbert Lagrangian as not physical in any direct sense. The co-existence of Lagrangians that do not differ merely by surface terms indeed suggests this view (espoused, for instance, by Brown & Holland, 2004, pp. 7). Then, independent criteria would be needed to make plausible the physical significance of the Einstein pseudotensor. Here, Pauli's considerations *would* be pertinent – and may be viewed as *disfavoring* the Einstein pseudotensor's suitability. But suppose, contrariwise, that one *did* consider the Lagrangian as something physical. For instance, the role the action plays, say, in the Feynman path integral or its link to black hole horizons (see e.g. Padmanabhan, 2005) might suggest such a realism.

But then it *would* matter whether one considered the full Einstein-Hilbert Lagrangian, its "ΓΓ"-version or perhaps a completely different Lagrangian – irrespective of their contribution to the field equations (or lack thereof) upon variation. Either way, Pauli's objection cannot be brushed aside lightly. (In Ch. IV, we shall see that canonical energy-momentum associated with the *full* Einstein-Hilbert Lagrangian, which thus includes also 2nd derivatives of the metric, suffers from additional problems of its own – grist to the mills of the sceptic of general-relativistic gravitational energy.)

Recently, Curiel (2014) closed this loop-hole. There exists indeed no tensor with the natural desiderata for representing gravity: apart from the Einstein tensor, no symmetric, divergence-free, homogeneous (for reasons of dimensionality) rank 2-tensor that vanishes if the spacetime is flat ($R^d_{abc} = 0$), can be constructed from up to second derivatives of the metric.

In fact, Lorentz and Levi-Civita proposed the Einstein tensor, $\mathrm{G}_{ab} = \mathrm{R}_{ab} - \frac{1}{2}R\mathrm{g}_{ab}$ (or, for reasons of dimensionality, $-\frac{1}{2\kappa}\mathrm{G}_{ab}$) as a suitable representation of gravitational energy (for a historical account of this proposal, see Pauli, 1981, fn. 350-351; Cattani & DeMaria, 1993, esp. sect. 5-11). We now turn to this proposal.

At first blush, it looks attractive. First, the Einstein tensor is a bona fide tensor. Secondly, it also obeys a bona fide covariant conservation law: the contracted Bianchi identity, $\nabla_b G^{ab} \equiv 0$. The attendant total energy-momentum ${}_{(LLC)}\mathfrak{T}^{ab} := -\frac{1}{2\kappa}G^{ab} + \mathrm{T}^{ab}$, satisfies both an ordinary and covariant continuity equation, $\partial_b\left({}_{(LLC)}\mathfrak{T}^{ab}\right) = \nabla_b\left({}_{(LLC)}\mathfrak{T}^{ab}\right) = 0$. Thirdly, the Einstein tensor is the exact *gravitational* counterpart of the *matter* energy-momentum tensor: whereas the latter is defined variationally as $T_{ab} = -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}\mathcal{L}_{(m)}\right)$, one obtains the Einstein tensor (up to a proportionality factor) by replacing the matter Lagrangian by the purely gravitational Einstein-Hilbert Lagrangian,

$$G_{ab} \propto \frac{1}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}R\right) \quad (11).$$

Two objections speak against the proposal: physical implausibility and vacuity, respectively. Firstly, consider the Einstein Equations in vacuum. This, on Lorentz and Levi-Civita's proposal, would yield *vanishing* gravitational energy, $G_{ab} = 0$. But that is counterintuitive: since the Einstein tensor is constructed from traces of the Riemann tensor, a solution of the vacuum

Einstein Equations has in general non-vanishing Weyl structure. The latter encapsulates gravitational radiation (see e.g. Padmanabhan, 2010, pp. 263) for technical details). Prima facie one would expect it to possess gravitational energy – contrary to Lorentz and Levi-Civita's proposal. Equally implausibly, it purports that there are no differences between gravitational energy in the exterior of a static and, say, charged rotating black hole, respectively: in either case, gravitational energy would be zero.

Besides doubts regarding its physical plausibility, it seems mysterious and contrived that, on Lorentz and Levi-Civita's proposal, any matter energy-momentum is exactly counterbalanced by gravitational energy. That is: in *all* possible spacetimes, the *total* energy always vanishes, $-\frac{1}{2\kappa}G_{ab}+2\kappa T_{ab}=0$. It is elusive what positing such an entity would help *explain*. In his correspondence with Levi-Civita, Einstein (1917, cited in op. cit., pp. 77) made this point. In a letter to him, Levi-Civita concedes that his proposal is indeed sterile in that "[…] the energy principle would lose all its heuristic value, because no physical process (or almost none) could be excluded a priori. In fact, [in order to get any physical process] one only has to associate with it a suitable change of the [gravitational field]".

In short: Lorentz and Levi-Civita's proposal lacks physical informativeness. The charge is aggravated by the fact that the contracted Bianchi identities, $\nabla_b G^{ab}\equiv 0$, as *mathematical* identities, barely count as conservation laws in any *substantive* sense. (By contrast, $\nabla_b T^{ab}=0$ requires a certain coupling of the metric to the matter fields. It thus hinges on physically substantive assumptions (for details, see Read et al., 2017 (ms), sect. 3.)

In consequence, Curiel's theorem seems to entail that local notions of gravitational energy will invariably conjure up non-tensoriality.[48] In the following, I will focus on pseudotensors, the most common type of non-tensorial objects.

[48] An interesting *tensorial* proposal, following Møller (reported in Goldberg, 1980, p. 477), is worth mentioning. For GR's truncated Einstein-Hilbert Lagrangian, Noether's theorem entails, as we'll see in §3.2, the conserved total current:

$$-2\sqrt{|g|}G^a_b\xi^b+t^{\ a}=\partial_b U^{[ab]},$$

where $\xi^m(x)$ is an arbitrary function. The *non*-tensorial Einstein pseudotensor – prima facie, the most natural candidate for representing gravitational energy – is obtained for *constant* ξ^ms, and the Einstein super-potential

$$U^{ns}=:\xi^m U_m{}^{ns}=2\sqrt{|g|}\xi^m\left(\delta^s_m\left\{\begin{matrix}[s\\ tr\end{matrix}\right\}g^{t]r}-\delta^n_m\left\{\begin{matrix}[s\\ tr\end{matrix}\right\}g^{t]r}+g^{r[n}\left\{\begin{matrix}s]\\ mr\end{matrix}\right\}\right)=:\sqrt{|g|}\xi^m\Xi^{ns}_m.$$

III.3.2. Pseudotensors

The Noetherian framework is the royal road to gravitational energy in GR. Field-theorists (e.g. Weinberg 1972) may feel inclined to treat GR like any other Lagrangian theory. Regarding the technical procedure, they may well be right (see the standard field-theoretical treatments in Horský & Novotný, 1969, or Barbashov & Nesterenko, 1983; for other possible advantages of their perspective, cf. Pitts, 2016ab). (I will ignore potential issues with diverging action integrals: the metric need not fall-off "nicely".) But interpreting the results is more subtle. Goldberg (1959, p. 319; cf. Horský & Novotný, 1969, pp. 427), for instance, observes: "Clearly, the existence of a complex with a vanishing divergence is insufficient evidence for the conservation of a physically interesting quantity." To this I now turn.

At this juncture, one introduces tetrads $\{\boldsymbol{e}_A = h_A{}^{\mu}\partial_\mu : A = 0,1,2,3\}$, i.e. a set of (orthonormal) basis vectors of the manifold's tangent space, $span\{\boldsymbol{e}_A : A\} = span\{\partial_\mu : \mu\} = T\mathcal{M}$ (for details, see e.g. Aldrovandi & Pereira, 2013, Ch. 1). Notice that $\{\partial_\mu\}$ here denotes a coordinate basis of tangent space (with Greek letters denoting spacetime indices in this footnote from now on). We can now express the Levi-Civita connection coefficients in terms of the tetrad components $h_A{}^{\mu}$ (see e.g. op.cit., pp. 7),

$$\left\{ {\mu \atop \nu\varrho} \right\} = h^A{}_{\varrho} \nabla_\nu h_A{}^{\mu},$$

the super-potential $U_\mu{}^{[\nu\sigma]}$ can be given a manifestly covariant form: $U_\mu{}^{[\nu\sigma]}$ is a (2,1)-tensor density of weight 1.

As for the Einstein pseudotensor $\xi^\mu = const.$, due to the anti-symmetry in the upper-indices the ordinary divergence of the Einstein super-potential still yields covariant term:

$$\partial_\sigma U^{\nu\sigma} = \sqrt{|g|}\nabla_\sigma\left(\xi^\mu \Xi_\mu^{[\nu\sigma]}\right) = \sqrt{|g|}\xi^\mu \nabla_\sigma \Xi_\mu^{[\nu\sigma]}.$$

Plugged back into the current equation above, this yields the Einstein pseudo-tensor:

$$t^{\,\nu} = \sqrt{|g|}\xi^\mu\left(2G_\mu^\nu + \nabla_\sigma \Xi_\mu^{[\nu\sigma]}\right).$$

This expression is manifestly covariant: it's a bona fide vector (of weight-1). Thanks to the transition to GR's tetradic formulation, we have thus overcome the pseudo-tensoriality! The Einstein gravitational energy-stress "pseudo"-tensor has become manifestly coordinate-independent.

This doesn't satisfactorily solve the problem of *general-relativistic* gravitational energy, though. First, tetradic GR and metric GR are arguably distinct theories: their solution spaces (with tetradic GR's being restricted to so-called parallelisable manifolds – which includes manifolds that have a Lie-group structure, but excludes e.g. all spheres other than $\mathbb{S}^1$, $\mathbb{S}^3$ and $\mathbb{S}^7$, or non-orientable manifolds, such as the Möbius strip) and referents (reference frames, represented by the set of tetrads, vs. spacetime's (chronogeo-)metric and inertial/affine structure) differ (cf. Combi & Romero, 2018).

Secondly, the tetradic approach flouts a form of GR's equivalence principle – "the equality of all (at least inertial) reference frames before the laws of physics": the laws of physics shouldn't privilege any reference frame (cf. Dieks, 1987). Indeed, locally (point-dependent) Poincaré transformed reference frames (represented by sets of tetrads) yield the same metric quantities (via $g^{\mu\nu} = \eta^{AB} h_A{}^{\mu} h_B^{\nu}$ with the (inverse of the) Minkowski metric η^{AB}). But $t^{\,\nu}$ isn't invariant under local Poincaré transformations – only under global (point-independent) Poincaré transformations! In this sense, the above approach establishes a well-defined/covariant gravitational energy – but at the price of (∞^6-times) empirically under-determined "sub-metric" (i.e. tetradic) surplus structure.

Can pseudo-tensorial expressions adequately represent gravitational energy-momentum 4-currents? With reason, one may challenge this. Two problems afflict pseudotensors: one is a problematic coordinate-dependence, the other a danger of arbitrariness/ad-hocness, due to ambiguity. While the second problem can be somewhat tempered, this comes at the price of trivialising the content of local gravitational energy-momentum.

The first problem stems from a formal property of pseudotensors: they are invariant only under affine transformations. Under more general transformations, pseudotensors are coordinate-dependent. Should this disconcert us? Not necessarily: Calling to mind the Kleinian conception of geometry, Wallace (2016) recently reiterated that nothing is *inherently* baneful about coordinate-dependent objects. For pseudotensors, however, the coordinate-dependence is "vicious" (Pitts): in general, the preferred coordinate transformations do *not* pick out the characteristic invariants of the spacetime. The spacetime symmetries do not align with (form at least a subgroup of) the pseudotensor's symmetry group – contrary to what one would expect of well-defined objects (Earman, 1989, Ch. 3.4).[49] This is highlighted by the fact that pseudotensors do not transform like 4-vectors neither under purely spatial transformations, $x^\mu \to x'^\mu = \left(x^0, x'^i(x^j)\right)$, "which mean nothing more than a mere renumbering of points of the three-dimensional configuration space" (Horkský & Novotný, 1969, p. 431), nor under purely temporal ones, $x^\mu \to x'^\mu = \left(x'^0(x^0), x^i\right)$, encoding "a continuous change in the rate and setting of the coordinate clock" (ibid.). In this sense, pseudotensors require structure absent in a given (non-flat) spacetime.[50]

In order both to connect the issue of pseudotensors with our thoughts from §II, and to prepare the discussion of another proposal in §3.3, it is rewarding to revisit two historical complaints about pseudotensors' coordinate-dependence (for a detailed account, see Cattani & DeMaria, 1993). Soon after Einstein's proposal of his pseudotensor, Schrödinger explicitly computed it for an incompressible fluid sphere. He showed that through a suitable choice of coordinates one can make the Einstein pseudotensor vanish. Einstein responded by showing that for systems of several masses, at least the Einstein pseudotensor cannot be made to vanish

[49] Given that the standard pseudotensors are built exclusively from spacetime-geometric quantities (the metric and its Levi-Civita connection), it's natural to expect the pseudotensor to represent a geometric feature. The pseudotensors' lack of invariance under general coordinate changes prevents this: of structures that are intrinsic to a geometry one demands such invariance.

[50] In Ch. IV, I'll elaborate on this: pseudotensors fail to be geometric objects in the formal sense of Anderson (1967, Ch. 1.5).

everywhere. Reversing Schrödinger's argument, Bauer subsequently observed that for suitable coordinates Einstein's pseudotensor also allows for even flat space possessing non-vanishing gravitational energy.

What to make of these objections? Let's look at Pitts' answers to them. According to Pitts (2010), Bauer failed to adapt his coordinates to the spacetime theories. But shouldn't the choice of non-adapted coordinates be irrelevant in *a generally covariant theory*? Isn't evaluating the gravitational energy-momentum flux $\chi_b j^b[\eta] \coloneqq \chi_b t^b_a \eta^a$ for some observer χ and along some direction η merely a question of *convenience*, not of principled importance? This casts Pitts' reply into doubt. For him, everything is as it should be – once one adopts privileged, *adapted* coordinates.

In §2.1, we identified the distinguished coordinates as inertial coordinates. In flat spacetime, these are globally Lorentzian. In them, the metric takes on a constant value everywhere; its derivatives vanish. Hence, the pseudotensor is indeed zero. So, Pitts is right in his counter to Bauer: adapting the coordinates to the symmetries of flat spacetime resolves Bauer's paradox.[51]

What does this reasoning imply for Schrödinger's objection, i.e. when applied to pseudotensors on *non-flat* spacetimes? From the definition of Einstein's pseudotensor, it is evident already that in inertial (i.e. Fermi or Riemann) coordinates it vanishes: Pitts' reply to Bauer thus *trivialises* the physical significance of the Einstein pseudotensor – which becomes simply zero!

Pitts gainsays this conclusion. According to him, the vanishing of the Einstein pseudotensor in, say, the exterior of a Schwarzschild black hole, when adopting uni-modular quasi-Cartesian coordinates, would be worrisome *only if* it could be made to vanish in a neighbourhood (thereby spoiling quasi-locality), or if the Einstein pseudotensor indeed vanished for every coordinate system. Neither is the case. More specifically, Pitts avers that there exist *infinitely many components* of gravitational energy (§3.3). Hence, according to Pitts, one need not be disquieted by the fact that *some* components are zero. This retort hinges crucially on Pitts'

[51] The same argument rebuts Read's remark that, on the (standard) response equation interpretation (see §2.1) even in flat spacetime $\nabla_a T^a_b = 0$, implies non-conservation of energy for arbitrary coordinates (see Read, 2017). In adapted/inertial coordinates of flat spacetime -viz. (global) Lorentz coordinates- $\nabla_a T^a_b = 0$ reduces to $\partial_a T^a_b = 0$.

own proposal for gravitational energy in GR. Reasons for scepticism about its adequacy will be presented in §3.3. Suppose here that the reader shares my scepticism. Then, the reasoning we reconstructed for Pitts' above reply to Bauer undermines his reply to Schrödinger: the adapted local inertial coordinates for the Schwarzschild case are indeed uni-modular quasi-Cartesian. In these coordinates, the Einstein pseudotensor is zero. (Inertial coordinates are adapted to free-fall frames: in them, the metric takes on a constant numerical value.)

Let us dwell a little on the vicious coordinate-dependence. It can become virulent also in practice – generalising Schrödinger's point. For instance, the Landau-Lifshitz pseudotensor – an alternative to Einstein's (see below) – yields *negative* energy densities for Reissner-Nordström spacetimes, when calculated in quasi-Cartesian coordinates (Virbhadra, 1991). Negative energy densities violate the weak energy condition.[52] Therefore, they are usually (e.g. Malament, 2012, Ch. 2.5) considered unphysical. By contrast, calculations of Einstein's and other pseudotensors give reasonable and mutually consistent results for Kerr-Schild Cartesian coordinates. For cylindrical gravitational waves, the pseudotensor exhibits a similar coordinate-dependence: their energy-momentum densities associated with the Einstein pseudotensor vanish in polar coordinates; Cartesian coordinates, by contrast, yield reasonable results (Rosen & Virbhadra, 1993). But what *exactly* disqualifies quasi-Cartesian coordinates? Kerr-Schild Cartesian coordinates are not adapted, *either*: they are not inertial coordinates. The same applies to the cylindrical gravitational wave: the global Cartesian coordinates employed there are not inertial.

The second problem of pseudotensors consists in their non-uniqueness (see Trautmann, 1962, esp. sect. 5-5; Anderson, 1967, Ch. 13 for details). An infinite number of possible alternative pseudo-tensors exists. None is a priori privileged over the other. We already saw that it does not affect the validity of a continuity equation, $\mathfrak{T}_a{}^b \coloneqq \sqrt{|g|}(T_a^b + \mathrm{t}_a^b)$, if we add an arbitrary superpotential of the form $-\partial_c \mathfrak{U}_a^{[bc]}$:

$$\partial_b \left(\mathfrak{T}_a{}^b - \partial_c \mathfrak{U}_a^{[bc]} \right) \equiv \partial_b \mathfrak{T}_a{}^b \quad (12).$$

Such an addition amounts to a re-distribution of total energy-momentum. Depending on how the metric falls off, this re-distribution is *physically significant* (especially when we consider

[52] That is: For a time-like vector field ξ and the energy-momentum-tensor T_{ab} the energy-density relative to ξ is positive: $T_{ab}\xi^a\xi^b \geq 0$

the associated integral/global quantities, i.e. energy-momentum proper – as opposed to the energy-momentum densities/fluxes). Note here two salient differences between GR and ordinary classical matter theories. First, the metric *never* has compact support (except, trivially, for compact manifolds); in fact, it vanishes nowhere. Classical matter, by contrast, typically (i.e. in most admissible physical models – and effectively *all* physically relevant ones) has compact support. Secondly and as a result, for the integrals over the above continuity equations to be even well-defined, certain *coordinate* conditions must obtain: else, the prerequisite fall-off behaviour of the metric components (and quantities derived from it) is violated. Given the conventionality of coordinate labels, the reliance on coordinate conditions is alarming (cf. Russell, 1927, Ch. VII). It stands in marked contrast to the situation of standard matter theories.

Via a choice of a superpotential and the Einstein Equations, one can define arbitrary pseudotensors:

$$\sqrt{|g|}t_a^{\ b} \coloneqq \partial_c \mathfrak{U}_a^{[bc]} + \frac{1}{2\kappa}\sqrt{|g|}G_a^b \quad (13).$$

Different choices of superpotentials correspond to different pseudotensors. Einstein's, for instance, follows from von Freud's choice of the superpotential,

$$_{(F)}\mathfrak{U}_a^{[bc]} = \frac{1}{2\kappa\sqrt{|g|}} g_{ad}\partial_e\left(|g|g^{b[d}g^{e]c}\right) \quad (14).$$

Is underdetermination the issue here? If so, wherein does the situation differ from the non-uniqueness of energy-momenta in other classical field theories? After all, they too are only defined up to a superpotential.

Consider the so-called "Bergmann form" of superpotentials:

$$_{(B)}\mathfrak{U}^{[ab]} := {}_{(F)}\mathfrak{U}_c^{[ab]}\xi^c \quad (15).$$

Here, ξ^a generates a one-parameter group of coordinate transformations whose variations, $\delta x^b = \epsilon\xi^b(x)$ leave the action (quasi-)invariant, with an infinitesimal ϵ. This one-parameter group forms a subgroup of the general continuous group of coordinate transformations. (In other words: we have a theory with local gauge symmetry[53] that allows for a non-trivial global subgroup. In this case, we can combine Noether's First and Second Theorem. For details, see e.g. Brown & Brading, 2000, sect.5; Brading, 2002, 2005; Ohanian, 2013.) Then, in vacuo ($T^a_b = 0$) one has purely gravitational (axial-/pseudo-vectorial) energy-momentum 4-current along ξ as

$$\sqrt{|g|}t^a[\xi] = \partial_b \left({}_{(F)}\mathfrak{U}^{[ab]}_c \xi^c \right) \quad (16).$$

In the presence of matter (with the associated 4-current $T^a_b \xi^b \neq 0$), we have the *total* energy-momentum 4-current along ξ,

$$\tilde{J}^a_{(tot)}[\xi] := \sqrt{|g|}(t^a + T^a_b \xi^b) \quad (17).$$

It can be re-written via a superpotential,

$$\tilde{J}^a_{(tot)}[\xi] = \partial_c \left({}_{(F)}\mathfrak{U}^{[ac]}_b \xi^b \right) \quad (18).$$

Due to the asymmetry in the superpotential's upper indices, the r.h.s satisfies an ordinary continuity equation in *all* coordinate systems $\partial_a \tilde{J}^a_{(tot)}[\xi] = 0$.

Consonant with our terminology of §2.1, the total-energy-momentum 4-current $j^a_{(tot)}$ possesses no sinks/sources.

Note that the quantities ξ^c need *not* constitute a vector field (Trautman, 1962). (In that case, formulating (16) or (18) in terms of a covariant derivative would no longer be well-defined.) E.g. choosing them such that the components $\frac{\xi^b g_{ab}}{\sqrt{|g|}}$ are constants yields an alternative to Einstein's, widespread in astrophysical applications (cf. Poisson & Will, 2016) – the Landau-Lifshitz pseudo-tensor ${}_{(LL)}\mathrm{t}^{ab}$. (It has the merits of being index-symmetric, ${}_{(LL)}\mathrm{t}^{[ab]} = 0$ and built from only 1st derivatives of the metric (Landau & Lifshitz, 1975, §96): ${}_{(LL)}\mathrm{t}^{ab} + \sqrt{|g|}T^{ab} = \partial_c \left(\sqrt{|g|} g^{ad} {}_{(F)}\mathfrak{U}^{[bc]}_d \right)$.[54])

[53] I use the term "gauge symmetry" here in the sense symmetries relating physically *identical* states of affairs – not in the sense of symmetries of gauge theories in the standard/Yang-Millsian sense (under which GR arguably cannot be classified, see e.g. Aldrovandi & Pereira, 2013, Ch. 3.3; Wallace, 2015 for details).

[54] Some authors, e.g. Ohanian & Ruffini, 2013, p. 493, fn 10, question the Landau-Lifshitz pseudotensor's physical significance. They see it compromised by its being a *weight 1*-density: the Landau-Lifshitz pseudotensor thus does not transform correctly under rigid coordinate transformations.

All known pseudotensors can be derived from the von Freud and Bergmann form, including Lorentz and Levi-Civita's proposal (for details, see Goldberg, 1958; Trautmann, 1962, p. 190; Horský & Novotný, 1969).

The freedom to choose a superpotential, with none intrinsically privileged, renders the local representation of gravitational energy *banefully* under-determined. There exist infinitely many superpotentials, one for each possible (not necessarily tensorial!) ξ^b. However, no such ξ^b is inherently privileged in a generic spacetime (with exceptions to be discussed presently): each corresponds to a possible "gauge" choice of translations. The gravitational energy-momentum 4-current $t^a[\xi] := \frac{1}{\sqrt{|g|}}\tilde{t}^a[\xi]$ *ad-hoc* privileges a direction. Equivalently, with each choice of a $t^a[\xi]$ one exalts – by an ad-hoc *stipulation* – a one-parameter group of coordinates transformations, by itself failing to be privileged.[55]

This is not merely a sin against GR's spirit. Different energy-momentum complexes *can* yield different energy distributions for the same gravitational background.[56] E.g. the energy for the exterior of the Kerr-Newman black hole, determined via Møller's pseudotensor, equals twice the energy, obtained from Tolman's, Einstein's or Landau/Lifshitz's pseudotensor (Virbhadra,

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[55] A complementary argument can be obtained from the Hamiltonian perspective. (Recall, however, that a Hamiltonian framework can't satisfactorily deal with typical dissipative systems – including gravitationally radiating ones.) Together with the Hamiltonian constraints and appropriate coordinate conditions, the Hamiltonian takes the form of the surface integral:

$$\mathcal{H} = -\frac{c^4}{16\pi G}\sum_{\alpha,\beta=1,2,3}\oint_{i^0} ds_\alpha\left(\partial_\beta h_{\alpha\beta} - \delta_{\alpha\beta}\sum_{\gamma=1,2,3} h_{\gamma\gamma}\right).$$

Here, ds_α denotes the surface element on spacelike infinity i^0 and $h_{\alpha\beta}$ is the spatial metric induced on the spacelike hypersurfaces of the 3+1-foliation (see e.g. Poisson, 2007, Ch. 4.2 for technical details).

Using different Hamiltonian formalisms, this surface integral can be represented in different ways, as volume integrals with different integrands:

- The standard ADM form:
  $$\mathcal{H}_{ADM} = -\frac{c^4}{16\pi G}\int d^3x \sum_{\alpha,\beta=1,2,3}\left(\partial_\alpha\partial_\beta h_{\alpha\beta} - \delta_{\alpha\beta}\sum_{\gamma=1,2,3} h_{\gamma\gamma}\right)$$
- Dirac's form:
  $$\mathcal{H}_D = \frac{c^4}{16\pi G}\int d^3x \sum_{\alpha,\beta=1,2,3}\partial_\alpha\left(|\gamma|^{-\frac{1}{2}}\partial_\beta\left(\gamma\gamma^{\alpha\beta}\right)\right),$$
  with $\gamma^{\alpha\beta}$ as the inverse of $h_{\alpha\beta}$ and $\gamma = \det\left(\gamma^{\alpha\beta}\right)$.
- Schwinger's form: $\mathcal{H}_S = \frac{c^4}{16\pi G}\int d^3x \sum_{\alpha,\beta=1,2,3}\partial_\alpha\partial_\beta\left(\gamma\gamma^{\alpha\beta}\right)$

Although the resultant total energies all agree, $\mathcal{H}_{ADM} = \mathcal{H}_D = \mathcal{H}_S$, the integrands, i.e. gravitational energy densities, differ *non-trivially*. Schäfer (2014, p. 17) concludes that "the notion of gravitational binding energy density has no physical or observational meaning."

[56] This does not seem to be the rule, though (Multamäki et al., 2008).

1990).[57] The ambiguity thus threatens the well-definedness of gravitational energy-momentum.

By contrast, the freedom in the choice of superpotentials is (in most cases) benign in pre-GR theories. Firstly, due to the compactness of the support of matter fields and suitable fall-off conditions, it doesn't affect the values of the corresponding Noether charges. Secondly, the spacetime settings of pre-GR theories contain symmetries. Their associated Killing vectors then serve as such a compass for privileged directions. I turn to this now.

One can evade the charge of ad-hoc privileging *arbitrary* directions by attending to those directions that *are* inherently privileged. (Recall §2.) Consider first *symmetric* spacetimes. Here, the directions along Killing field ξ^b are privileged.

For such spacetimes, Komar (1959) arrived at the following expression for the superpotential:

$$_{K}\mathfrak{U}^{[ab]}[\xi] = \frac{1}{2\kappa}\partial_b\left(\sqrt{|g|}\nabla^{[a}\xi^{b]}\right) \quad (19).$$

The resulting total 4-current (weight-1 density) reads:

$$\tilde{J}^a_{(tot)} = \sqrt{|g|}\left(t^a[\xi] + T^a_b\xi^b\right) = \partial_b\left({}_{K}\mathfrak{U}^{[ab]}\right) \quad (20).$$

Thanks to its anti-symmetry, Komar's superpotential can be re-written explicitly as a genuine tensor density of weight one:

$$\partial_b\left(\sqrt{|g|}\nabla^{[a}\xi^{b]}\right) = \sqrt{|g|}\nabla_b\left(\nabla^{[a}\xi^{b]}\right) \quad (21).$$

Consequently, $J^a_{(tot)} = \frac{1}{\sqrt{|g|}}\tilde{J}^a_{(tot)}$ is indeed a genuine vector. It is covariantly conserved:

$$0 = \partial_a\tilde{J}^a_{(tot)} = \sqrt{|g|}\nabla_a J^a_{(tot)} \quad (22).$$

Given that $\nabla_a(T^a_b\xi^b) = 0$, it follows that also the gravitational energy-momentum 4-current $t^a[\xi]$, too, is a genuine vector that is covariantly conserved, $\nabla_a t^a[\xi] = 0$.

[57] The energy distributions of the Einstein and Møller pseudotensor differ also for the deSitter, the Schwarzschild solution, the charged regular metric, the stringy charged black hole and Gödel-type spacetimes (Gad, 2004, p. 2).

We can evaluate the gravitational 4-current,

$$t^a[\xi] = \frac{1}{\sqrt{|g|}}\left(\partial_b\left({}_K\mathfrak{U}^{[ab]}[\xi]\right) - \sqrt{|g|}T^a_b\xi^b\right) \quad (23)$$

by harnessing the Killing property $\nabla^{(a}\xi^{b)} = 0$:

$$t^a[\xi] = -\frac{1}{\kappa}\Box\xi^a - T^a_b\xi^b \quad (24).$$

Here, $\Box := g_{ab}\nabla^a\nabla^b$ denotes the d'Alembert operator.

However, neither term making up this 4-current is suitably connected with gravitational degrees of freedom to represent local gravitational energy-momentum. Consider first the second term, $T^a_b\xi^b$. It is the (conserved) matter energy-momentum flux along the direction of the Killing field. As such, it is unrelated to gravitational degrees of freedom.

The first term, $\Box\xi^a$ *is* related to gravitational degrees of freedom via the identity, holding for all Killing fields (e.g. Padmanabhan, 2010, p. 220)

$$\nabla_b\nabla_a\xi_c = R_{dbac}\xi^d \quad (25).$$

Contraction yields

$$\Box\xi^a = R^a_b\xi^b \quad (26).$$

So, albeit indeed related to gravitational effects (taken here to be represented by curvature effects), the first term is too *coarsely* related to gravitational effects: in particular, it ascribes to *all* matter-free regions of any arbitrary spacetime the same value: zero. This seems counterintuitive. (Notice that with (25), the r.h.s. of equation (24) is nothing but $-\frac{1}{\kappa}(R^a_b + \kappa T^a_b)\xi^b$. Hence, due to the Einstein Equations, $t^a[\xi] \equiv 0$.)

In conclusion, for symmetric spacetimes, Komar's candidate for gravitational energy-momentum turns out not to be related in the right way to gravitational degrees of freedom.

For *non-symmetric* spacetimes, the only inherently privileged ξs are those describing inertial trajectories. In analogy to the ontological status of apparent forces, I argued earlier that for energy-momentum balances we should be realists only about those terms that survive in inertial frames $*$ – in particular upon switching to normal coordinates. In the adapted/comoving (i.e. normal) coordinates, the Bergmann form trivially vanishes. (Recall:

$g|_* = const.$) Harking back to our thoughts in §2.1, we educe that the resulting gravitational energy-momentum 4-current *worthy of realist commitment* is zero:

$$t^a[\xi]|_* = 0 \quad (27).$$

In summary, for both cases where the arbitrariness objection to pseudotensors could seemingly be averted, we wind up with the same conclusion as in §2.1-2: gravitational energy-momentum is trivialised. With the problem of vicious coordinate-dependence still looming without remedy, it thus seems preferable to reject the pseudotensorial approach to local gravitational energy-momentum altogether.
I will therefore move on and inspect two heterodox alternative proposals from the more recent philosophical literature.

III.3. Pitts' object

Recently, Pitts (2010) made an astute suggestion: take your pet pseudotensor, and declare the totality of its values in every possible coordinate system an infinite-component object sui generis, with each component corresponding to the value of the pseudotensor in some coordinate system. Since each component satisfies a continuity equation, so does the whole object (suitably defining derivatives for such objects).

Pitts' object provides thoughtful answers to the criticism of §2.2: by construction, it is (in a suitable sense) coordinate-*independent*. Hence, it extricates gravitational energy-momentum from vicious coordinate-dependence. Pitts rightly extols this.

What about the ambiguity/arbitrariness problem? It persists. If one picks *one* pseudotensor of the Goldberg-Bergmann type and "Pittsifies" it, one obtains indeed a well-defined object. Yet, why prefer *this* pseudotensor over others? In terms of the Von Freud or Bergmann form, different choices for a preferred direction for a gravitational energy-momentum are still possible. Hence, one can construct again an infinite number of Pitts objects, one for each pseudotensor. Furthermore, why not Pittsify other, non-pseudotensorial expressions (involving e.g. background metrics or auxiliary connections, each in itself no less suitable a priori)? This exacerbates the ambiguity. (Normally, one could plausibly discard such objects as parasitic on auxiliary structure that GR simpliciter lacks. Pitts' strategy deprives one of this argument: in a straightforward formal sense, the object Pittsified over, say, all possible auxiliary metrics *no longer* depends on this auxiliary structure; only each coordinate/component (associated with each particular background metric) does. In consequence, in order to restrict Pittsification to pseudotensors, Pitts has to summon other arguments than he has presented so-far.)

He might parry by demonstrating that one particular Pitts object, say, the Pittsified Belinfante-Rosenfeld symmetrised Einstein pseudotensor, is indeed the best candidate. This is certainly *conceivable*: the list of attractive pseudotensors of the Bergmann form can be further whittled down by excluding e.g. the Landau-Lifshitz pseudotensors or the Møller pseudotensor (on account of its anomalous factor, diagnosed by Katz (1985)). To-date, though, such a comprehensive analysis is still pending.

Should our hopes for uniqueness be dashed, Pitts (2017, sect. 13.4) envisages a way to turn this vice into a virtue: as the action contains infinitely many symmetries, it may appear natural to allow for infinitely many gravitational energies. Perhaps, Pitts proposes, this proves an advantage in the context of black hole thermodynamics: after all, Nester and collaborators suggested that different gravitational energies correspond to different free energies and the like under different boundary conditions. However, to judge that such considerations buttress Pitts' proposal seems premature: at present, it is controversial (see e.g. Dougherty & Callender, 2016; cf. however Wallace, 2017a, Appendix) whether the correspondence between black hole thermodynamics and thermodynamics is substantive, rather than a speculation based on partial and formal analogies.

So, let's assume that the (non-)uniqueness problem defies a satisfactory resolution. In that case, Pitts (2017, p. 270) rightly warns against double standards: one must not demand of gravitational energy, what *non*-gravitational energy does not satisfy, either. Pitts points out that non-uniqueness poses a problem even for scalar fields (Callan et al., 1970). GR's gravitational energy (Pitts-style) would then appear no worse off than other field theories. But this, I think, is misleading: attempts to improve the (Belinfante-Rosenfeld-symmetrised) canonical energy-momentum tensor, "[…] are largely 'ad hoc' procedures focused on special models of field theory, often geared to the needs of quantum field theory and ungeometric in spirit" (Forger & Römer, 2003, p.3). Requiring a certain "ultra-locality", Forger and Römer show in a geometric, systematic manner that uniqueness of the energy-momentum tensor *can* be restored: for non-gravitational matter, it coincides with the variationally defined energy-momentum tensor. (For its gravitational counterpart, the Einstein tensor, this leads us back to Lorenz and Levi-Civita's proposal of §3.1.) Of course, Forger and Römer's result can scarcely lay claim to a proof from indubitable first principles. If thus one regards it as little more than a stipulation, it would be unfair to decry Pitts' proposal as wanting for its inability to solve the uniqueness problem: *nobody* who champions some realism for gravitational energy in GR has solved it. The main thrust of my critique of Pitts' proposal in this section then could therefore only be twofold. First, to question some of Pitts' claims that his proposal provides any particular *advantage*. This might eventually turn out to be true, but at present, more work needs to be done. Secondly, I happily grant Pitts' proposal the status of the most promising avenues for realists about gravitational energy. Hence, if even *it* suffers from grave problems – so much the better for eliminativism.

I close with two considerations, both revolving around the physical significance of Pitts' object. First: Doesn't Pitts owe us an argument why his object should be considered physically *meaningful*? Analogously, we could Pittsify, say, the electromagnetic potentials of a system: gather the totality of all its possible gauges into one formal object. Is this artificial seeming Pittsified 4-potential physically meaningful, or even useful? I am not sure. By the same token, I am not aware of any theoretical or practical context in which physicists ever calculate infinite numbers of energies.

Pitts could respond: *wherever* gravitational pseudotensors are useful, his proposal affords a coherent interpretation of such pseudotensors. Let's consider three more concrete forms of this argument, related to an interpretation to the Noether Theorems, the equivalence with the Einstein Equations, and Anderson's framework of geometric objects, respectively.

The first can be cashed out as the ability of Pitts' strategy to provide an intelligible interpretation of Noether's Theorems for GR. It gives what seems the natural answer to the question of how many conserved energies there are in GR, namely: infinitely many – corresponding to infinitely many possible rigid time translation symmetries. But the argument is not entirely compelling: why should the Noether Theorems be in *need* of a physical interpretation? Whether one thinks they do, depends on one's willingness to regard the Lagrangian as physical. This is controversial. Arguably, it is more natural to regard the Lagrangian merely as a computational prop. The Noether machinery is only a tool to conveniently derive continuity equations. Their validity, however, does not presuppose anything from the Noetherian framework: they follow from the field equations alone.[58]

A second variant of the argument focuses on the interpretation of these continuity equations. Pitts reminds us of Anderson's observation that the totality of continuity equations of all possible pseudotensors is equivalent to the Einstein Equations: surely, one may be tempted to conclude, since the latter are physically meaningful, so is the equivalent totality of continuity equations, i.e. the local conservation law for the corresponding Pitts object. At least for the time being, I want to resist that temptation. Should one infer from the equivalence of Feynman's path integral formalism with standard quantum mechanics that their paths equipped with their *complex* amplitudes provide a coherent interpretation of quantum mechanics? This would be too quick (Zeh, 2011). Likewise, one may challenge that a concept's computational utility on its own suffices to warrant the kind of realism that underlies Pitts' reasoning. Compare: It is at least controversial whether to include the quantum potential amongst the ontology of Bohmian Mechanics – despite its utility in many applications, such as the semi-classical approximation schemes (see e.g. Goldstein, 2017, sect. 5 for details).

The third and last worry about the significance of Pitts' object consists in some discomfort one may feel about the expressly non-geometric nature of Pitts' object: how does such a non-geometric and non-tensorially representable object fit into a geometric, local field theory? Pitts replies: if Anderson's standard formal framework of geometric objects (with a finite number of components) cannot accommodate gravitational energy, but the latter is a good idea, seek an alternative framework that encompasses also non-geometric objects! To me it seems more cautious to resolve this conflict in favour of the framework of geometric objects: If forced to choose between a well-established, useful "global" ontological framework and my hunches about *one* quantity whose meaningfulness has conceptually already been called into question, I prefer to sacrifice the latter.

[58] To the extent that Pittsification is touted as a natural reading of Noether's (First) Theorem, another challenge looms. As Brown (2020) emphasises, the modern version of Noether's theorem doesn't establish a one-to-one map between (quasi-)symmetries in the action and conservation laws, "but between suitably identified equivalence classes of both" (ibid. p. 10, see also for further details of this equivalence class). Hence, a "natural reading of Noether's Theorem" should yield *full* Pittsification, i.e. realism about an equivalence class of Pittsified objects. (Here, the equivalence class would consist of all Pitts objects that belong to the same equivalence class of conservation laws that figure in the modern Noether map.) This further quotienting out may further increase the impression that Pitts' proposal is tainted by artificiality.

In summary: While interesting and promising, Pitts' object leaves questions crucial for its physical significance unanswered.

III.3.4. The cosmological constant

According to its standard interpretation, one construes the cosmological constant Λ as vacuum energy. Based on this standard interpretation, Baker (2004, sect. 4.2) raised the following question: if Λ is the energy contribution from empty space-time, shouldn't $T_{ab}^{(\Lambda)} := -\frac{\Lambda}{2\kappa}g_{ab}$ count as a natural candidate for gravitational energy-density? Not only does $T_{ab}^{(\Lambda)}$ play the functional role of a (negative) energy density of a perfect "cosmic fluid", composed purely of matter-free spacetime, but due to metric compatibility, it even satisfies a conservation law, $\nabla^b T_{ab}^{(\Lambda)} = 0$.

Three reasons militate against Baker's proposal. First, the Einstein Equations with a cosmological constant, $G_{ab} + \Lambda g_{ab} = 2\kappa T_{ab}\kappa$, are a minimal *extension* of Einstein's original GR, obtained from adding a constant to the Einstein-Hilbert Lagrangian. Hence, for standard GR, in which the cosmological constant is absent/zero, the gravitational energy would identically vanish. This trivialises gravitational energy. Secondly, what is *really* meant by interpreting Λ as vacuum energy? On one common view, this vacuum energy refers to the sum of the energy fluctuations of the quantum mechanical ground state.[59] Λ thus is *not* energy of "empty space". Rather, it is the zero-point energy of an all-pervasive quantum field, i.e. attributable to *matter*. Lastly, although $T_{ab}^{(\Lambda)}$ plays the functional role of an energy density, nothing compels us ascribe it to the r.h.s. of the Einstein Equations, as a source: no less would we be licenced to ascribe it to the l.h.s., where Λ could simply serve as some parameter of the "gravitational field strength functional"[60], ${}_{\Lambda}G_{ab}[g_{ab}] := G_{ab} + \Lambda g_{ab}$ – a status comparable

[59] This interpretation of Λ displays a discrepancy between the quantum field theoretical predictions and cosmological observations by roughly 120 (!) orders of magnitude (see e.g. Carroll, 2000; cf. Rugh & Zinkernagel, 2000).
But even as a non-fundamental/effective, purely phenomenological description the cosmological constant admits of an interpretation as a matter parameter, characterising a perfect cosmic fluid with equation of state parameter $w = \frac{p}{\varrho} = -1$.

[60] By that I mean the following. Construe both sides of the Einstein Equations as functionals of the metric (as well as, for the energy-momentum tensor, the matter fields), $G_{ab}[g_{ab}] = 2\kappa T_{ab}[g_{ab};\Psi]$. (Recall that T_{ab} depends on g_{ab}, see Lehmkuhl, 2010 for an analysis.) The Einstein Equations codify how the matter fields and the metric interdepend (Nerlich, 2013, Ch. 9): together with initial data, they determine (some part of) the dynamics of the metric. (Recall that the Einstein Equations *constrain*, but don't *fix* the trace-free part of the Riemann tensor, the Weyl tensor.) The functional on the r.h.s., $T_{ab}[g_{ab};\Psi]$, plays the role of a source density, analogously to the

with parameters featuring in other non-linear theories, e.g. the soliton equation. Should one indeed interpret Λ as a property of spacetime itself, then via a contraction of the Einstein Equations (in vacuum) it is easily seen to equal scalar *curvature* of spacetime, unperturbed by matter: $\Lambda = 4\mathrm{R}$. This contradicts what one would expect of a suitable notion for gravitational energy: it's too coarse-grained; it would not yield any (intuitively expected) contributions from gravitational radiation (encoded in Weyl structure, a part of the Riemann curvature not fixed by the Einstein Equations, see e.g. Padmanabhan, 2010, Ch. 5.3.3). Furthermore, the proposal does not reduce to the potential energy of Newtonian gravity.

In conclusion, at best, Baker's interpretation of Λ as gravitational energy lacks plausibility; at worst, it rests on a conflation of classical and quantum vacuum.

III.4. Summary and conclusion

We started the preceding analysis by investigating conservation of energy-momentum in generic, non-symmetric spacetimes. Considerations of the priority and explanatory distinction of inertial frames led us to restrict our realist commitments in energy-momentum balances to the terms retained after an evaluation in inertial frames and the adapted coordinates. We found that the matter energy-momentum 4-currents along the inherently preferred directions in such non-symmetric spacetimes (viz. along inertial trajectories) possess no sinks/sources. Whilst thus matter energy-momentum is conserved locally, *globally* it is not: the energy-momentum contained in an observer's spacelike hypersurface varies in time.

In symmetric spacetimes, the matter energy-momentum 4-current along their Killing fields is conserved both locally *and* globally: the associated "charges" are independent of the choice of space-like hypersurfaces.

Non-trivial local gravitational energy-momentum did not arise in these considerations: it turned out to be an idle wheel in the context of matter energy-momentum conservation. This inspired the working hypothesis that gravitational energy-momentum is eliminated in GR:

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particle current, $J_A^a := \frac{\delta I_p}{\delta A_A^a}$, (say, fermions) of a Yang-Mills field A_A^a and interaction Lagrangian I_p, (Szabados, 2012, sect. 3.1.2). The functional on the l.h.s., $G_{ab}[g_{ab}]$ plays the role of a field-strength functional, with the metric as the "field strength".

analogously to apparent forces in CM, it is reduced to representational artefacts conjured up by physically unprivileged descriptions.

Subsequently, we scrutinised proposals for local *gravitational* energy-momentum. Nontensorial expressions are inevitable. The discussion focused on pseudotensors, as they emerge naturally from generalisations of Noether's Theorems, applied to GR's purely gravitational Lagrangian. Pseudotensors face two main problems: first, a mismatch between the spacetime symmetries and the symmetries that their preferred coordinates pick out, and secondly an ambiguity that threatens to introduce arbitrariness/ad-hocness. The latter worry can be allayed – but only at the price of trivialising the gravitational energy-momentum 4-current. These issues suggest that one abandon also the pseudotensorial route to local gravitational energy-momentum.

As a way out, we considered Pitts' strategy to define an infinitely many component object, via the totality of all possible pseudotensors. Whilst addressing the issue of vicious coordinate-dependence, Pitts' proposal provided no satisfactory answer to the problem of arbitrariness. Its physical significance remains doubtful.

Eventually, we examined and discarded Baker's proposal of the cosmological constant as a candidate for local gravitational energy.

GR forces us to revise (i.a.) two notions, central to pre-GR hunches. First, global energy-momentum conservation becomes a contingent fact, dependent on the contingent symmetries of the spacetime. Secondly, local gravitational energy-momentum is eliminated: it is no longer a meaningful physical quantity.[61] In a sense, it has been geometrised (or rather: "inertialised") away.

This verdict is largely in agreement with the "orthodox" GR literature (e.g. Pauli, 1981, p. 177; Weyl, 1923, p. 273; Eddington, 1923, p. 137). Even Einstein (1918, in: Gorelik, 2002, p. 25), despite initially advocating his pseudotensorial approach, ultimately conceded that "(t)hus […] we come to ascribe more reality to an integral than to its differentials." Others (e.g. Weyl, 1923, p.273; Wald, 1984, p. 70, fn 6) concurred: can we reserve a realist commitment only for

[61] NB: Stephani (2004, Ch. 28.4) goes even a step further: he disputes that energy, gravitational or otherwise, ceases to be fundamentally meaningful in GR. Accordingly, the issue of its conservation becomes moot. I baulk at such a radical conclusion, confining eliminativism to a more conservative eliminativism about gravitational energy alone.

the *global* (integral) notions of gravitational energy-momentum? But this is a question for another chapter.

This chapter:

We considered and rejected various proposals for local representations for gravitational energy: locally, gravitational energy – like the gravitational force – seems indeed to have been “geometrised” (or rather: “inertialised”) away. Conservation of non-gravitational matter seems to hinge on whether the spacetime has symmetries.

The next chapter:

Can we formulate the challenges for local gravitational energy a bit more precisely? And what about global notions? Albeit perhaps not fundamental, could gravitational energy be meaningfully defined as a higher-level concept (like “neuron” or “anteater”), useful and worthy of a realism of sorts under certain circumstances?

IV. Against ‘Functional Gravitational Energy’

Abstract:

This chapter revisits the debate between realists about gravitational energy in GR (who opine that gravitational energy can be said to meaningfully exist in GR) and anti-realists/eliminativists (who deny this). In particular, I’ll extend the previous chapter’s analysis to global/integral notions of gravitational energy. I re-assess the arguments underpinning Hoefer’s seminal eliminativist stance, and those of their realist detractors’ responses. A more circumspect reading of the former is proffered that discloses where the so far not fully appreciated, *real* challenges lie for realism about gravitational energy. I subsequently turn to Lam and Read’s recent proposals for such a realism. Their arguments are critically examined. Special attention is devoted to the adequacy of Read’s appeals to functionalism, imported from the philosophy of mind.

Key words: gravitational energy, energy conservation, General Relativity, functionalism, idealisations vs. approximations, GR-exceptionalism

IV.1. Introduction

This chapter scrutinises Read's recent claim that a so-called functionalist strategy can support realism about gravitational energy in General Relativity (GR) − the view that GR possesses local and global gravitational energy-stress in a robust physical sense (at least within a certain class of models). According to Read (2018), such a realism chimes with, and sanctions the use of, gravitational energy common in astrophysical practice.

Read's arguments, I'll argue, aren't convincing – at least not as they stand. In particular, not only is gravitational energy explanatorily dispensable in GR – as Read admits. It's, I submit, a tenuous explanans.

In what follows, I pursue three goals: 1. to critique Read's proposal, 2. to plead for anti-realism about gravitational energy, and 3. to animadvert upon facile uses of functionalism.

The first goal is to push back against Read's realism about gravitational energy: even if one is sympathetic to his overall general argument, one may well question that gravitational energy satisfies its premises.

My second goal is indirect: I'll take up the cudgels for Hoefer's eliminativism about gravitational energy (Hoefer, 2000). Hoefer's arguments admit of a different, more circumspect formulation. Thus re-formulated, they evade Read's objections. This also brings to the fore the more serious difficulties that realists about gravitational energy face. I'll argue that Hoefer's eliminativism is ultimately a more satisfactory stance than Read's.

A third goal is to enhance our understanding of functionalism in the philosophy of physics. §4 provides a critical discussion of a recent application of functionalism −viz. Read's. This allows us to demarcate more sharply (or, at least, sensitivises us to pitfalls with respect to) what functionalist strategies can and can't achieve – and what suitable contexts for their application *might* be.

A recurrent theme will be a major, yet somewhat underappreciated question in the extant GR literature:[62] to what extent is GR special in comparison to, say, electromagnetism or Yang-

[62] The most notable and insightful exceptions are found in the oeuvre of Pitts, esp. (2016ab, 2017, 2018). Kaiser (1998) reviews the historical dialectics between GR exceptionalism and egalitarianism between 1942 and 1975.

Mills theories? Advocates of an "egalitarian" view downplay this specialness (e.g. Feynman, 1995; or Brown, 2005). They can rightly point to the – at least, occasional − fertility of such a position. (Think, for instance, of spin-2 derivations of the Einstein Equations, see e.g. Pitts, 2016c, or the clarification of the *mis*interpretation of the cosmological constant as a mass term, Pitts, 2019.) Likewise, egalitarians must be given credit for often exposing double standards frequently applied to GR. Contrariwise, advocates of "exceptionalism" affirm GR's privileged status. They accentuate the distinguished explanatory (and, plausibly attendant, ontological) status of GR's spacetime structure (see e.g. Janssen, 2009 (whom I take to extend his views on Special Relativity to GR); Nerlich, 2007; 2013). Standard GR is indeed special in some regards: conceptually-formally, it differs from our best fundamental theories of matter, i.e. Yang-Mills gauge theories (see Weinstein, 1999; Aldrovandi & Pereira, 2013, Ch. 3.3; Teh, 2014; Wallace, 2018 for details). One major difference is that GR's fundamental variable is a metric – not a connection. From the fibre bundle perspective, another major difference consists in GR's display of so-called soldering: internal ("gauge") and external ("spacetime") degrees are inextricably linked. (It lies outside of the present thesis' scope to explore the implications and interpretative significance of these peculiarities of general-relativistic gravity.)

One's realist/anti-realist attitudes towards gravitational energy in GR tend to be fuelled by egalitarian/exceptionalist presuppositions. Usually, they remain implicit. It lies outside the present chapter's ambit to adjudicate between GR egalitarianism and GR exceptionalism. Instead, I'll flag where one's verdict on the force of certain (counter-)arguments in the realism/anti-realism debate about gravitational energy hinges on such prior commitments. In this regard, I'll advance the following claim: in terms of realism/antirealism about gravitational energy, Read's functionalism brings nothing new to the table which transcends the realism one may *already* cherish towards gravitational energy. *Only* if one is already attracted to realism about gravitational energy (undergirded by egalitarianism about GR), will one find Read's position attractive, too. It doesn't furnish, however, any independent arguments for such a realism.

The chapter will proceed as follows. In **§2**, I'll introduce the dispute between realists about gravitational energy, such as Read, and antirealists, such as Hoefer. The next section, **§3**, hones in on Hoefer's arguments (**§3.1**), and Read's responses to them (**§3.2**). Both are assessed in **§3.3**. **§4** is devoted to Read's own realist proposal. I'll first, in **§4.1**, reconstruct the logical structure of his argument. **§4.2** critically evaluates it. Finally, in **§5**, I'll outline two contexts not considered by Read: non-tensorial global notions of gravitational energy, and Ashtekar's asymptotics programme.

IV.2. Setting the stage: realism about gravitational energy

In this section, I'll delineate the tenets of realism about gravitational energy, as they appear in the debate between Read (who advocates it) and Hoefer (who discards it).

The debate revolves around the following conundrum. GR's field equations can be derived from varying the Einstein-Hilbert action[63]

$$\int d^4x\sqrt{|g|}\left(\mathcal{L}_{(g)}+\mathcal{L}_{(m)}\right).$$

The purely gravitational Lagrangian $\mathcal{L}_{(g)}$ is given by the Ricci scalar, $\mathcal{L}_{(g)}=R$, or the so-called, dynamically equivalent (e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 19) "$\Gamma\Gamma$" Lagrangian $\overline{\mathcal{L}}_{(g)}=2g^{\mu\nu}\Gamma^{\lambda}_{\mu[\nu}\Gamma^{\kappa}_{\lambda]\kappa}$. (It has the advantage of being only first order, in analogy with Lagrangians of other field theories.) $\mathcal{L}_{(m)}$ denotes the matter Lagrangian.

Applying Noether's 1st Theorem (or a suitable generalisation – what Brown and Brading call the "Boundary Theorem") to this total Lagrangian density, $\mathcal{L}_{(m)}+\overline{\mathcal{L}}_{(g)}$, yields a continuity equation (see e.g. Barbashov & Nesterenko, 1983 for details):

$$\partial_b\left(\sqrt{|g|}(T_a^{\ b}+\vartheta_a^{\ b})\right)=0.$$

Here, $T_a^{\ b}$ denotes the energy-stress tensor associated with ordinary matter fields. $\vartheta_a^{\ b}$ is the canonical energy-momentum associated with the purely gravitational Lagrangian density $\overline{\mathcal{L}}_{(g)}$. It's dubbed the Einstein pseudotensor,

$$\vartheta_a^{\ b}=\frac{1}{\sqrt{|g|}}\left(-\bar{\mathcal{L}}\delta_a^b+\frac{\partial\bar{\mathcal{L}}}{\partial(\partial_b g_{de})}\partial_a g_{de}\right).$$

[63] For technical details, see e.g. Poisson (2004), Ch.4.

It transforms tensorially only under affine transformations. Hence, its qualifier "pseudo". Despite the Einstein pseudotensor's non-tensorial nature, the above continuity holds for all coordinate systems.

Given the exactly analogous construction as canonical energy-momentum in other field theories, the Einstein pseudotensor is naturally construed as a local (differential) gravitational energy-stress density. (Below, I'll suppress "density" for the sake of readability.)

In consequence, the above continuity equation is naturally interpreted as local conservation of total energy-stress: total (gravitational *plus* matter/non-gravitational) energy-stress, $\mathcal{T}_a^b := T_a^b + \vartheta_a{}^b$, has neither sources nor sinks.

Henceforth, I'll refer to this interpretation of the pseudotensor as ''realism about *local* gravitational energy-stress" ***(REAL$_{LOC}$)***. I'll treat it as concomitant with the interpretation of $T_a^b + \vartheta_a{}^b$ as local total energy-stress.

Via an application of Gauß's Theorem, one may now try to convert the continuity equation into a conserved *global* (integral) quantity over a 4-volume. For the integrals to be well-defined, certain conditions must hold. Preliminarily, I'll subsume them under the label "asymptotic flatness". More details will follow in §2.1.

The view that this quantity denotes *global* total energy-stress and is conserved – attendant with the view that the integral over the pseudotensor denotes global gravitational energy-stress- will be referred to as "realism about global gravitational energy-stress and energy-stress conservation" **(*REAL$_{GLOB}$*)**.

This position is strictly weaker than its local counterpart. Provided one counts a possibly divergent integral as a well-defined, but infinite quantity, one can advocate (*REAL$_{GLOB}$*) without (*REAL$_{LOC}$*), but not vice versa.[64] (Such a situation is familiar from other areas. Think, for instance, of entropy production in thermodynamics. In, say, a Carnot cycle, entropy production $\delta Q_{rev}/T$, with the reversible heat energy transfer δQ_{rev} and temperature T, is defined only up to "thermal gauge transformations", i.e. exact one-forms. Hence, "locally" entropy isn't well-defined. Only "globally" it is, i.e. the integral $\int \delta Q_{rev}/T$. For a field-

[64] One may regard this, i.e. (*REAL$_{GLOB}$*)&¬ (*REAL$_{LOC}$*), as the orthodox position: Several early authors (e.g. Einstein, 1918, cited in: Gorelik, 2002, p.25; Eddington, 1923, p.137; Pauli, 1918, §61; Weyl, 1921, §31; Schrödinger, 1950, p. 100); Misner, Thorne & Wheeler, 1972, §19-20) reserved realism for global gravitational and total energy-stress.

theoretical example, think of the self-current of Yang-Mills theories, $j^{A\mu} = -f^{A}_{BC}A^{B}{}_{\lambda}f^{C\lambda\mu}$. Being explicitly dependent on the connection $A^{B}{}_{\lambda}$, it's gauge-variant. By contrast, due to the sourceless Yang-Mills equations, $\partial_{\nu}F^{A\mu\nu} = j^{A\mu}$ (with $F^{A\mu\nu}$ denoting the curvature of the connection), the "charge" $Q^{A\mu} = \int d^4x\sqrt{|\eta|}\, j^{A\mu}$ is gauge-invariant.)

Hoefer and Read disagree over whether or not to adopt realism about (local and/or global) gravitational stress-energy. Read affirms both ($REAL_{GLOB}$) and ($REAL_{LOC}$) in certain contexts; Hoefer opposes them without qualification. What are their respective arguments?

IV.3. Hoefer's eliminativism – Read's response

In this section, I'll first review a straightforward reconstruction of Hoefer's objections to gravitational energy, together with his rejoinder that in GR energy conservation should be abandoned (§3.1). Subsequently (§3.2), I'll inspect Read's responses. They are critically evaluated in §3.3.

The analysis will cast into sharper relief the *real* problems that (Read's) realism about gravitational energy must address. They'll play a pivotal role in §4.2.

IV.3.1 Hoefer's eliminativism

In favour of eliminativism about gravitational energy and energy conservation, ¬($REAL_{GLOB}$) & ¬($REAL_{LOC}$), Hoefer mounts three arguments: *(H1)* coordinate-dependence, *(H2)* ambiguity, and *(H3)* inapplicability of the conditions that the definability of gravitational energy presupposes.

Hoefer's first point, *(H1)*, is that realism about energy-stress conservation "goes against the most important and philosophically progressive approach to spacetime physics: that of downplaying coordinate-dependent notions and effects, and stressing the intrinsic, covariant and coordinate-independent as what is important" (pp. 194).

According to Hoefer, the pseudotensor featuring in ($REAL_{LOC}$) doesn't comply with this precept: "its non-tensorial nature means that there is no well-defined intrinsic 'amount of stuff' present at any given point" (ibid.). Neither does ($REAL_{GLOB}$) comply - presumably because asymptotic flatness, as Hoefer presents it, is formulated via the following *coordinate* conditions:

(a) For $r := \sqrt{x^2 + y^2 + z^2} \to \infty$, the coordinate system must be asymptotically Lorentzian, i.e. $g_{\mu\nu} \to \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. In the interior, it can vary arbitrarily.

(b) The metric must decay sufficiently rapidly: $g_{\mu\nu} \to \eta_{\mu\nu} + \mathcal{O}(1/r)$, $\partial g_{\mu\nu} \to \mathcal{O}(1/r^2)$, $\partial^2 g_{\mu\nu} \to \mathcal{O}(1/r^3)$.

Hoefer's second objection, *(H2)*, (ibid.) attacks the pseudotensor's ambiguity: it's not uniquely defined. Some elaboration is in order of what Hoefer may have had in mind. The pseudotensors are defined only up to a transformation of the form ${\vartheta_a}^b \to {\vartheta_a}^b + \partial_c \Xi_a^{[bc]}$. Here, $\Xi_a^{[bc]}$ is a so-called superpotential, anti-symmetric in its upper indices (see Trautman, 1965, for details). As a result it's vastly underdetermined, thereby impeding (*REAL$_{LOC}$*).

Hoefer's third argument, *(H3)*, targets (*REAL$_{GLOB}$*). His thought seems to be that (*REAL$_{GLOB}$*) hinges on realism about the conditions under which it's well-defined, i.e. asymptotic flatness. Hoefer correctly observes that our actual world isn't asymptotically flat. Realism about asymptotic flatness thus is mistaken. This, according to Hoefer, undercuts (*REAL$_{GLOB}$*).

Hoefer's point straightforwardly carries over to (*REAL$_{LOC}$*). In flat spacetimes, pseudotensors are to *some extent* extricated from their unsettling non-tensorial transformation behaviour: in them, Poincaré transformations – i.e. at least a subgroup of linear transformations – *are* distinguished as relating physically equivalent frames. Alas, Hoefer might interject: our universe isn't flat – not even asymptotically. An advocate of (*REAL$_{LOC}$*) thus has to stomach non-tensoriality.

Based on his diagnosis of coordinate-dependence, ambiguity and anti-realism about the formal prerequisites for defining global gravitational energy, Hoefer champions antirealism/eliminativism about gravitational energy: we should relinquish the notion. Instead, we should just accept that in GR, energy conservation no longer holds.

How does Read respond to Hoefer's arguments?

IV.3.2. Read's response

While for Hoefer the above reasons suggest that one abandon (*REAL$_{GLOB}$*) and (*REAL$_{LOC}$*), Read wants to resist this conclusion. He parries by *(R1)* rejecting Hoefer's ban on coordinate-based language, *(R2)* by assuming that the non-uniqueness can be overcome (or, at least, isn't problematic), *(R3)* by defending the use of idealisations, and *(R4)* by baulking at the revisionary nature of Hoefer's eliminativism.

First, Read takes Hoefer to outlaw the usage of coordinate-dependent notions *(H1)*. Read rightly repudiates this as unwarranted *(R1)*. The mere usage of coordinates is unproblematic:[65] "[…] presentations of spacetime theories need not proceed in a coordinate-independent manner; rather, spacetime theories may be defined in terms of equations written in a coordinate basis and their transformation properties (this is what Brown […] and Wallace […] refer to as the 'Kleinian conception of geometry'), and explanations may be given by appeal to those laws, written in a coordinate basis." On this Kleinian conception, one characterises geometry via the class of privileged coordinate systems (see Wallace, 2016 for details). In these, the dynamical equations preserve a particular (e.g. simplest) form. Such coordinate-based characterisations are as coordinate-*in*dependent as those not based on coordinates, i.e. drawing on intrinsically geometric notions. Prima facie, Read thus effectively wards off Hoefer's first complaint.

It might even appear that *(H1)* was unfounded for ($REAL_{GLOB}$) from the outset: while it's popular and expedient to define asymptotic flatness in a coordinate-based manner (e.g. Jaramillo & Gourgoulhon, 2010 for a more detailed presentation), this *isn't* necessary. Via conformal techniques, it's indeed possible to characterise asymptotic flatness in purely geometric, coordinate-free terms (e.g. Geroch, 1972, Ch. 35-38; Wald, 1984, Ch. 11; Ludvigsen, 1999, Ch. 12) – as Hoefer demands.

To Hoefer's complaint of the ambiguity of pseudotensors *(H2)*, Read responds as follows *(R2):* "There are many distinct but non-equivalent choices for this pseudotensor, based on one's choice of superpotential. Hence […] we are implicitly supposing that a choice has been made from the family of possible candidates" (p. 11). (Below, I'll also consider a different response that Read may be read as endorsing.)

In his third response, *(R3)*, Read rebuts Hoefer's attack on asymptotic flatness as an assumption not applicable to our universe *(H3)*. Read acknowledges: it is "[…] undeniable […]

[65] Geometric/coordinate-free formulations are even ill-suited for applications of GR's initial value problem (see, e.g., Isenberg, 2014).

that the entire universe is not asymptotically Minkowski" (pp. 16). Yet, according to Read, asymptotic flatness is a good idealisation for certain approximately isolated subsystems.[66]

Read rightly underscores that "every theory of physics is an idealisation and does not 'apply to the actual world' in this strong sense" (p. 17). He takes Hoefer to reject asymptotic flatness as an *ultimately* inaccurate assumption. That, however, Read argues, demands too much of successful hypotheses for them to earn realist commitments: ultimate exactness is *never* attainable. Rather, Read suggests that this doesn't curtail the utility of asymptotic flatness as an idealisation.

Read's final response, *(R4)*, is to avoid Hoefer's eliminativism due to its "potentially undesirable consequences". On the one hand *(R4a)*, "such a claim would also commit one to the statement that there exists no genuine stress-energy conservation law in [Special Relativity, SR] – a theory in which the conservation of total stress-energy typically is taken to be uncontroversial" (p. 18). On the other hand *(R4b)*, "the advocate of the Hoefer-type view is apparently committed to the denial of the claim that gravitational waves and other forms of purely gravitational radiation are energetic". Read avers that this is gratuitously revisionary.

Do Read's responses −the legitimacy of coordinate-based language, the implicit supposition that the non-uniqueness can be overcome, the legitimacy of asymptotic flatness as an approximation, and the rebarbative ramifications of Hoefer's eliminativism− effectively rebut Hoefer's worries? In the next paragraph, I'll assess Read's answers.

IV.3.3 Hoefer reloaded

I'll now critically examine Read's counters to Hoefer, *(R1)-(R4)*. Each, I submit, misses the more subtle points of Hoefer's critique: *(R1)* conflates the mere usage of coordinates with a vicious coordinate-*dependence*; *(R2)* merely voices a *hope*, not an argument; *(R3)* ignores the

[66] Characterising the domain of applicability of asymptotic flatness as "systems within the world […] considered in isolation" (Read, 2018, p. 17) verges on being tautological (cf. Curiel, 2000, pp. 17): one *defines* a general-relativistic system as (materially and gravitationally) isolated *because* its total energy content is conserved; otherwise, one would regard it as (at least) gravitationally interacting.
To avoid this vacuity, I take Read to make the more specific claim that certain subsystems of the universe that don't interact *non*-gravitationally are approximately asymptotically flat.
To jump ahead a little: the preceding claim can't be universally true −as witnessed by textbook FLRW cosmologies: their matter sector is modelled by cosmic dust, i.e. a homogeneous, isotropic fluid with negligible non-gravitational interactions, see e.g. Hobson, Lasenby & Efstathiou, 2006, Ch. 14. One therefore ought to understand Read's claim as this: there exists a physically relevant, and empirically well-corroborated class of only gravitationally interacting systems that are asymptotically flat. We'll return to this in §4.

distinction between approximations and idealisations; *(R4)* is in part, both exegetically and systematically unwarranted, and in part an appeal to majority consensus.

Let's begin with *(R1)*, Read's rehabilitation of coordinate-based descriptions à la Klein. I deem it a red-herring: it's an infelicity in Hoefer's presentation of his argument which invites the misunderstanding that Hoefer wishes to ban coordinate-based language per se. A more disconcerting *real* issue lurks behind his worry, though: pseudotensors, which figure in *(REAL$_{LOC}$)*, are artefacts of conventions; something akin besets the integral over them.

Read is certainly right in that neither coordinate-*relativity* nor non-tensoriality need prevent us from ascribing an object physical significance. The Levi-Civita connection coefficients, $\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left(\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}\right)$, attest to that: geometrically, they define a privileged (viz. geodesic) path-structure (equivalently, in the language of fibre bundles: they connect the fibres of the tangent bundle over different points of the base manifold); physically, they encode inertial structure.

Yet, pseudotensors are plagued by "*vicious* coordinate-dependence" (Pitts[67], 2009, p. 16): they pick out preferred coordinates in the above Kleinian sense that *don't* align with the spacetime symmetries. Equations involving pseudotensors preserve their invariance only under affine coordinate transformations. But what distinguishes them in non-flat spacetimes? (Recall: In generic spacetimes affine coordinate transformations *aren't* preferred in the Kleinian sense.) Conversely, how to make sense of the fact that pseudotensors don't respect spacetime symmetries? That is, how to understand the fact that their invariance isn't preserved under spacetime symmetry transformations – in contradistinction to what one expects of matter fields (cf. Pooley, 2013)?

These oddities are highlighted by the fact that pseudotensorial 4-fluxes of gravitational energy-momentum, $\vartheta_{\mu}^{\ \nu}\xi^{\mu}$ (along the direction of ξ) *don't* transform like 4-vectors under purely spatial, or under purely temporal transformations. But both amount to a *merely conventional* re-labelling of points in space, and continuous change in the rate and setting of a coordinate clock (Horský & Novotný, 1969, p. 431), respectively. Neither should impact physical quantities – such as energy-momentum fluxes.

[67] Pitts propounds a method to construct geometric, infinite-component objects out of pseudotensors (see below). Those overcome the vicious coordinate dependence – at least formally.

By stressing merely their non-tensoriality, Read downplays the viciousness of the dependence of the pseudotensors: they aren't merely (and, as the Kleinian hastens to add: benignly) "frame-relative"; they are viciously frame-*dependent*. This has its exact counterpart in the frame-dependence of non-invariant quantities in SR: the latter don't represent objective features of the world; reifying them leads to the notorious "paradoxes" of SR (see Maudlin, 2011, Ch. 2; 2012, Ch. 4 for lucid explications).

It's instructive to re-phrase the problem: pseudotensors don't form geometric objects in the sense of e.g. Anderson (1965, Ch. 4.13, 1967).[68] A geometric object y on an *N*-dimensional manifold $\mathcal{M}$ is a correspondence $y: \langle p, \{x^\mu\}\rangle \rightarrow \langle y_1, \dots, y_N\rangle \in \mathbb{R}^N$ which associates with every point $p \in \mathcal{M}$ and every local coordinate system $\{x^\mu\}$ around p an *N*-tuple $y \coloneqq \langle y_1, \dots, y_N\rangle$ of real numbers (the object's components), together with a definite transformation rule that relates the components relative to the original coordinate system, and the components $y' \coloneqq \langle y'_1, \dots, y'_N\rangle$ relative to a different coordinate system $\{x'^\mu\}$ around *p*. The transformation rule only involves the object's components y, y' relative to the coordinate systems $\{x^\mu\}$ and $\{x'^\mu\}$, and their Jacobi matrix $\partial x/\partial x'$.

Requiring that the transformation rule depend only on the the components and the Jacobi matrix (i.e. y, y' and $\partial x'/\partial x$) is necessary for the mutual consistency of legitimate coordinate systems in the following sense: whenever we can use different coordinate systems, the order in which we switch from one to the other them doesn't matter. This can be stated more precisely. Let the transformation rule $T[y, y', \partial y/\partial x, \partial x/\partial x']$) for the coordinate transformation $\{x^\mu\} \rightarrow \{x'^\mu\}$ depend on, say, $\partial y/\partial x$. Consider now a $\{x''^\mu\}$ is a third coordinate system. Suppose that relative to it, $T[y', y'', \partial y'/\partial x', \partial x'/\partial x'']$) also holds. In general, it won't follow, however, that $T[y, y'', \partial y/\partial x, \partial x/\partial x'']$) is satisfied (for details, see Kucharzewski, M. & Kuczma, M., 1964).

In short: The components of geometric objects in arbitrary coordinates are uniquely determined by their components in one coordinate system and the transformations between the coordinates.

[68] The notion of geometric object objects was standard in differential geometry from the 1930s onward. (It shouldn't be conflated with Anderson's related programme of analysing substantive general covariance in terms of *absolute* objects, cf. e.g. Pitts, 2006.)

By way of example, note that connection coefficients $\Gamma^{\mu}_{\kappa\lambda}$ *form* a geometric object. Under coordinate changes, they transform as

$$\Gamma^{\mu}_{\kappa\lambda} \rightarrow \Gamma'^{\mu}_{\kappa\lambda} = \frac{\partial x'^{\mu}}{\partial x^{\varrho}} \frac{\partial x^{\sigma}}{\partial x'^{\kappa}} \frac{\partial x^{\tau}}{\partial x'^{\lambda}} \Gamma^{\varrho}_{\sigma\tau} + \frac{\partial^2 x^{\sigma}}{\partial x'^{\kappa} \partial x'^{\lambda}} \frac{\partial x'^{\mu}}{\partial x^{\sigma}}.$$

By contrast, consider the vector field v^{μ}. Then, the quantity $\frac{\partial v^{\mu}}{\partial x^{\nu}}$ *doesn't* form a geometric object: under coordinate changes, it transforms as

$$\frac{\partial v^{\mu}}{\partial x^{\nu}} \rightarrow \frac{\partial v'^{\mu}}{\partial x'^{\nu}} = \frac{\partial x'^{\mu}}{\partial x^{\kappa}} \frac{\partial x^{\lambda}}{\partial x'^{\nu}} \frac{\partial v^{\kappa}}{\partial x^{\lambda}} + \frac{\partial x^{\lambda}}{\partial x'^{\nu}} \frac{\partial^2 x'^{\mu}}{\partial x^{\kappa} \partial x^{\lambda}} v^{\kappa}.$$

That is: The transformation rule exhibits the prohibited dependence on v^{κ} (rather than $\frac{\partial v^{\kappa}}{\partial x^{\lambda}}$). One can indeed straightforwardly verify that in virtue of this dependence, successively applying the preceding transformation law to two coordinate transformations, $\{x^{\mu}\} \rightarrow \{x'^{\mu}\}$ and subsequent $\{x'^{\mu}\} \rightarrow \{x''^{\mu}\}$ yields a transformation law, *different* from the one for the coordinate transformation $\{x^{\mu}\} \rightarrow \{x''^{\mu}\}$.

In the same sense, being non-geometric objects, pseudotensors are viciously coordinate-dependent: the transformation rules of pseudotensors – if they are well-defined at all – exhibit a dependence on the coordinates employed.[69] The consistency condition is violated.

Geometric objects, however, constitute the standard framework within which physical objects of contemporary field theories are couched (see Nijenhuis, 1952; Schouten, 1954; Anderson, 1967, 1971; Torretti, 1996, Ch. 4.3).[70] They ensure that the intrinsic properties of physical entities and all relations between them are preserved under general coordinate transformations – the mere re-labelling of the manifold points. "Thus the components of a geometric object form a natural kind mathematically: they constitute faces of one and the same entity by virtue of being interrelated by a coordinate transformation law" (Pitts, 2009, p. 610). (Note that this is compatible with the existence of special coordinates, in which the physical laws take a particularly simple form.) By contradistinction, the properties and relations of entities represented by non-geometric objects are, as it were, sensitive to the labels attached to spacetime points. But such labels are usually deemed merely conventional.

[69] NB: Generically each pseudotensor has a different (viciously coordinate-dependent) transformation rule.

[70] It's worthwhile mentioning that Friedmann (1983), in contrast to the cited authors, restricts geometric objects to either tensors or connections. Thereby he –unduly – neglects, for instance, tensor densitities of arbitrary weight or projective connexions (cf. Pitts, 2006; 2012; Schouten, 1954, p. 301).

(Equivalently: Non-geometric objects presuppose more structure – information encoded directly in coordinates of the manifold points – than a manifold, standardly construed, contains.) Due to their non-geometric nature, the physical significance of pseudotensors, and hence the tenability of *(REAL$_{LOC}$)*, thus becomes questionable.

To be sure, Read could stand by his guns: he might withdraw his allegiance to the geometric object programme.[71] Suppose that a pseudotensor ${\theta_a}^b[\langle\tau, \Sigma_\tau\rangle]$ is only meaningful relative to a *given* coordinate system. Let the latter represent a (3+1)-decomposition ("frame"), $\langle\tau, \Sigma_\tau\rangle$. Relative to a different frame, $\langle\tau', {\Sigma'}_\tau\rangle$ one obtains a distinct object, ${\theta'_a}^b[\langle\tau', {\Sigma'}_\tau\rangle] \coloneqq {\theta_a}^b[\langle\tau', {\Sigma'}_\tau\rangle]$. Vicious coordinate dependence of ${\theta_a}^b$ now has lost its sting: ${\theta_a}^b$and ${\theta'_a}^b$ represent distinct entities.

What impedes the interpretation of such frame-relative objects is that no (3+1)-decomposition is distinguished over any other. To preserve this "frame-egalitarianism", one has two options. The first one is to *renounce* realism about those ${\theta_a}^b[\langle\tau, \Sigma_\tau\rangle]$s for *all* possible frames. This is tantamount to anti-realism towards pseudotensors. The second option is to *extend* one's realist attitude to *every* ${\theta_a}^b[\langle\tau, \Sigma_\tau\rangle]$ for all frames. The idea is to lump the totality of all $Q[\langle\tau, \Sigma_\tau\rangle]$s for all possible frames, $\{\langle\tau, \Sigma_\tau\rangle\}$, into one formal object - symbolically:

$$\Theta \coloneqq \{{\theta_a}^b[\langle\tau, \Sigma_\tau\rangle] : \forall\langle\tau, \Sigma_\tau\rangle\}.$$

(Think of each ${\theta_a}^b[\langle\tau, \Sigma_\tau\rangle]$ as one of the uncountably infinitely many components of Θ.) A realist about Θ doesn't privilege any of its components. Thereby, she respects frame-egalitarianism. (Θ is even a geometric object in a slightly relaxed sense.[72]) Pitts (2009) has indeed made this astute proposal.

[71] Contra Read's (2018, §3.1) remark, absent any explicit discussion in his work (to my knowledge), it's hard to say – and possibly a rewarding reconstructive task, integrating his views on coordinates (see e.g. Norton, 1989, 2002) and interpretation of GR (see e.g. Lehmkuhl, 2014) – whether Einstein himself would have had qualms about non-geometric objects. While he repeatedly objected to the requirement that all meaningful objects be tensorial, that view is, as we saw, compatible with an insistence on geometric objects. Indeed, Torretti (1996, p. 316, fn1) views Einstein's insistence on a "*definite* transformation rule" as essentially an endorsement the geometric object framework. (This is compatible with the passage, cited by Read (fn 22), in which Einstein defends his pseudotensor against his colleagues' complaint. Therein, Einstein (1918, p. 449) critiques their view that "all physically significant quantities can be understood as scalars and tensor components" (my translation). It's not clear, however, that Einstein fully understood the non-geometric nature of his pseudotensor (avant la lettre). Yet, later on (p. 452) in the cited text, Einstein seems to concede some unease about his pseudotensor: "[…] we thus come to ascribe more reality to the integral than to its differentials" (my translation). I thank James Read for pressing me on this.)

[72] Usually (e.g. Trautman, 1965, p. 85 or Anderson, 1967, p. 15) one considers only geometric objects with a *finite* number of components.

Here, we needn't arbitrate between the anti-realist first, or the realist second option. It's clear, however, that at this stage *($REAL_{LOC}$)* is staked on the plausibility of Pitts' proposal. Whether the latter is persuasive remains to be seen (cf. Curiel, 2018, fn 27; Dürr, 2018a, §3.3, i.e. Ch. III.3.3 of this thesis, for a critique). Read, anyway, stays silent on the matter. (Plausibly, a defence of realism about Pitts' proposal deploys a double strategy akin to Read's: 1. to appeal to scientific utility to licence realist stance towards it, and 2. to appeal to similarities with pre-relativistic notions of gravitational energy in order to identify Pitts' object as their genuine, general-relativistic analogue. Read's crucial −to-date unaccomplished− task would then be to flesh all of this out in detail.)

In the same vein, Read's Kleinian vindication of coordinate-use doesn't allay a related worry for *($REAL_{GLOB}$)*. Different coordinate choices can give rise to different (or even ill-defined) distributions of global (gravitational) energy-stress (see Xulu, 2003, for a survey of explicit calculations). To maintain realism about pseudotensor-based integral quantities, one must cope with the ambiguity resulting from such coordinate-dependence.

There are three options. The first is to remove the ambiguity by privileging certain coordinate systems (e.g. quasi-Cartesian ones). This seems to contravene frame-egalitarianism. (On the other hand, Read might argue – as does e.g. Pitts (2010) – that our world, to a good approximation, does privilege quasi-Cartesian coordinates anyway. But first, one may worry whether such an appeal to approximate symmetries is sufficiently robust: *how* good need the approximation be for it to legitimately privilege quasi-Cartesian coordinates? Secondly, as we'll see below (§4.3.2), quasi-Cartesian coordinates aren't privileged, when applied to the universe as a whole – nor generic subsystems. If, however, one restricts oneself to not too large spacetime regions that can be approximated as roughly Minkowskian, quasi-Cartesian coordinates are indeed privileged for those subsystem. But such a quasi-Minkowksian regime is contingent, and fairly arbitrarily stipulated: what then justifies Read in distinguishing it for characterising gravitational energy and/or energy conservation?)

The other two options are in line with frame-egalitarianism. One is to adopt anti-realism about such integral quantities. This defeats Read's realist ambitions. He should therefore pursue the third option – the integral/global version of Pitts' proposal: *all* integral quantities are real. That is, he should adopt realism about infinite-component objects of the (quasi-symbolic) type

$$\int \Theta \coloneqq \left\{ \int dV \theta_a{}^b \left[\langle \tau, \Sigma_\tau \rangle \right] : \forall \langle \tau, \Sigma_\tau \rangle \right\}.$$

As before in the case of *(REAL$_{LOC}$)*, the plausibility of Read's version of *(REAL$_{GLOB}$)* hinges on the plausibility of realism towards such objects. Again, Read's position doesn't add a new argument for *(REAL$_{GLOB}$)*. Rather, it crucially relies on a prior commitment to a realism about the integral Pitts-object – for which Read gives no argument.

In summary: A conservative framework for classical field theories is Anderson's geometric objects programme. Within it, a physical interpretation of pseudotensors, as envisaged by *(REAL$_{LOC}$)*, is doubtful: they aren't geometric objects. *(REAL$_{GLOB}$)* fares no better: pseudotensor-based global notions of gravitational energy are coordinate-dependent artefacts of conventions. Read has two options: either to reject (or revise) the geometric object framework, or to extend his realism to Pitts' object. For either choice, we are owed an argument. These problems hold *irrespective* of one's predilection for a Kleinian or a coordinate-free approach to geometry. Read's response *(R1)* cuts no ice against them**.**

Let's continue with *(R2)*, Read's second response, regarding the pseudotensor's ambiguity *(H2)*: he *simply assumes* that one can learn to live with the plurality, or that uniqueness can be restored in a principled manner.

The ambiguity of pseudotensors bodes ill for *(REAL$_{GLOB}$)*: different pseudotensors can also yield different global energy distributions (see again Xulu, 2003, also for further references).

One response is, of course, to accept the ambiguity about gravitational energy-stress. But such pluralism has a drastic conclusion: via the First Law of Thermodynamics, it threatens to subvert the uniqueness of thermodynamic states more generally. Read shies away from this (pers.comm.).

Read's hopes should therefore be set on a way of *coping with* the non-uniqueness. But he remains silent on *how* to achieve this. Why believe Read's "implicit supposition" *(R2)?*

Two possible reasons spring to mind. One is that perhaps uniqueness can be restored; the other is to bite the bullet: perhaps the non-uniqueness is a feature, not a bug.

It's certainly *conceivable* that the list of viable pseudotensors can be further whittled down. For instance, vis-à-vis its anomalous factor, it's plausible to exclude the Møller pseudotensor (Katz, 1985). More general arguments for a unique expression are collated in works by Katz

(2005), Katz, Bičak & Lynden-Bell (2007) and Petrov (2008). (Note that these authors use a background metric. Vis-à-vis such auxiliary structure one may already ponder: does its introduction compromise the result?) While an enticing project, a comprehensive analysis of such an agenda is pending.

Another possibility for coping with the non-uniqueness is "to try to find meaning in it" (Pitts). In this spirit, Pitts (2017), following Nester (2004) and collaborators, suggests that the pseudotensors' ambiguity is a blessing in disguise: their differences correspond to different free energies and the like under different boundary conditions. It remains to be seen whether this proposal is convincing (cf. Dürr, 2018a, p.11). At present, it too is an enticing project, not a clear-cut argument in Read's favour.

In short: As it stands, Read's response (R2) falls short of being an argument. At present, whether uniqueness for gravitational energy can be restored is an open question. Likewise, whether non-uniqueness is an advantage, remains controversial.

Read's third response *(R3)* takes Hoefer to reject asymptotic flatness for not applying to our universe. Read seeks to legitimise its use as an approximation. This way of portraying Hoefer's criticism, however –as a demand for *excessive* rigour – glosses over a deeper concern: to what extent may we assume that the universe possesses the relevant structures that gravitational energy presupposes?

We can render the question's import more transparent by dint of Norton's distinction between approximations and idealisations (Norton, 2012). The former denotes an inexact description of the target system. The approximation's referent coincides with it. An idealisation, by contrast, is an (inexact) description of a surrogate system that mimics the target system in relevant regards. An idealisation's referent is thus *distinct* from the target system.

Given a supremely successful model, an inference to the best explanation (IBE) entails different realist stances towards it −depending on whether one classifies it as an approximation or an idealisation (cf. ibid, §2.4; Torretti, 1990, Ch. 3.6). In the first case, an IBE licences realism about the model totaliter: the target system can be assumed to *actually* possess, at least roughly, the properties of the model. By contradistinction, an IBE about an idealisation licences only a "*selective* realism": we may only assume that the target object shares *some* structural features with the model – those responsible and indispensable for the model's explanatory success, its "working posits" (see e.g. Vickers, 2016; 2017 for details). Only they −*not* the model tout court− merit a realist interpretation.

Norton's distinction affords a refined reading of Hoefer's objection to asymptotic flatness: rather than an intolerably imprecise approximation, asymptotic flatness is an idealisation of our actual world. Even when successful, an IBE about asymptotically flat models consequently doesn't warrant an *unqualified* realism: only their working posits merit realism. Hoefer's criticism *(H3)* is thus best construed as the view that asymptotic flatness is an *idle* posit of an idealisation. By contrast, Read's response *(R3)* is more sanguine: asymptotic flatness is either an approximation, or a *working* posit of an idealisation. Hence, it literally (albeit only in approximation) depicts real structures in the world. Neither Hoefer nor Read proffers arguments for their respective verdicts, thus construed. I'll arbitrate between them in §4.2.

To summarise: Hoefer's objection to asymptotic flatness is best interpreted as the view that the explanatory successes of relativistic astrophysics and cosmology don't justify the belief in approximate asymptotic flatness. Read champions this belief. Neither backs up his stance by arguments.

With his fourth response, *(R4)*, Read goes on the offensive. He points to two unappealing alleged consequences of Hoefer's position. Neither argument strikes me as cogent.

Read's first claim, *(R4a)*, is that Hoefer's eliminativism implies also a failure of energy-conservation in Special Relativity (SR) − provided one demands that an acceptable conservation principle hold in *every* frame. I agree with Read about this *conditional* claim. But I find no textual evidence that Hoefer endorses the antecedent condition.[73] But even if he did: his eliminativism isn't inherently tied to the (implausible) doctrine that conservation principle must hold in all frames.[74] In fact, for the same reasons why fictitious forces in Classical Mechanics (e.g. the Coriolis force) are artefacts of descriptions in ill-adapted, *generic*

[73] The two passages that *might* suggest the contrary are the following. In the first one, (p. 193), Hoefer says about the validity of the pseudotensor-based continuity equation in all frames that "[…] (it) can be taken as generally covariant." This suggests that he equates general covariance with validity in arbitrary coordinate-systems. But in light of the Kretschmann argument (e.g. Norton, 1985, sect. 5, whom Hoefer cites!), this sense of general covariance is merely formal: any theory can be made to conform to it.
The second passage is along similar lines. Hoefer (p. 195) approvingly quotes Stephani: the latter laments the non-tensoriality of pseudotensorial gravitational energy as a violation of general covariance.
[74] This doctrine is arguably a relic of Einstein's (erroneous) initial understanding of general covariance (see again Norton, 1985 for details).

coordinate systems that needn't disturb us (see e.g. Maudlin, 2012, pp. 23, fn. 7), we needn't be worried by a conservation principle formulated in special coordinate systems.

If we thus drop the doctrine of equality of all frames, SR's conservation law remains untouched – as Read (2018, sect. 2.4) admits: owing to the existence of a time-like Killing field in Minkowski spacetime, a bona fide, tensorial local and global conservation law is straightforward (e.g. Straumann, 2013, Ch. 3.4). The coordinates adapted to these symmetries are the familiar (globally defined) Lorentz coordinates. In them the matter energy-stress tensor satisfies an ordinary continuity equation, with its standard interpretation.

Read's second claim, *(R4b)*, is that on Hoefer's view, the standard interpretation of binary systems must be jettisoned: in this account, gravitational energy evidently is a central explanans (e.g. Hobson, Efsthathiou, Lasenby, 2006, Ch. 18). An eliminativist about gravitational energy, however, abjures it. Read's lesson: So much the worse for Hoefer's eliminativism. But Read's argument is an argumentum ad verecundiam: it merely cites orthodoxy in the physics community. What are the cogent, *systematic* reasons to subscribe to it? I'll return to this in §4.

In short, neither claim comprising *(R4)* is persuasive: *(R4a)* implausibly imputes to Hoefer an implausible doctrine; *(R4b)* is an appeal to majority.

The insights gained here will pave the ground for our discussion of Read's own position – the topic of our next section.

IV.4. Functional Gravitational Energy and its Discontent

Here, I'll first (§4.1) lay out Read's functionalist approach to gravitational energy. Its logical structure will be made explicit. Subsequently, (§4.2) I'll critically examine three of its crucial premises. I reject them for multiple reasons. Notwithstanding my sympathies to his overall functional approach, and to the Dennettian ontological framework, I conclude that Read's realism should be renounced.

IV.4.1 Functional Gravitational Energy

Here, I'll expound Read's realism about gravitational energy-stress (Read, 2018, §3.3.2, §3.3.3), and the logical structure of his argument for it. Read proposes to embrace the background relativity of gravitational stress-energy (in the sense of §3.3). As this background-relative notion is both useful, and satisfies the functional role of gravitational energy, according to Read, we should be realists about it.

By "background" Read (and Lam, see below) mean (asymptotic) symmetries, encapsulated in asymptotic Killing fields, and suitable fall-off conditions, both implemented via asymptotic flatness. Lam and Read suggest that one should regard local and global gravitational and total energy as quantities well-defined relative to this background.

Let's unravel his reasoning in more detail. Read picks up an earlier intimation by Lam (2011): on the one hand, "[…] within [GR] all meaningful notions of (gravitational and nongravitational) energy-momentum […] require the introduction of some background structures" (p. 1023); on the other hand, if these structures are present, genuine gravitational and non-gravitational energy exists: "they make only sense in particular (but very useful) settings" (ibid.).

Read's realism, *(REAL$_{LOC}$)* & *(REAL$_{GLOB}$)*, can now be cashed out as positive, principled answers to the following two questions (p. 19):

(a) Does the pseudotensor ${\vartheta_a}^b$ in *(REAL$_{LOC}$)* and its associated integral ("charge") in *(REAL$_{GLOB}$)* represent anything real? Are these *formal* terms grounded in physical (but not necessarily fundamental) quantities?

(b) Suppose a positive answer to (a). Are we then licenced to identify the quantities that ${\vartheta_a}^b$ and its associated charge represent as gravitational energy-stress? "(I)s it correct to call the quantity appearing in [the continuity equation of *(REAL$_{LOC}$)* and its integral form in *(REAL$_{GLOB}$)*] […] 'gravitational stress-energy'"?

The questions in (a) require a reality criterion. Echoing Lam, Read appeals to the explanatory and predictive utility of the gravitational pseudotensor and its associated charge: "[…] (they) are only well defined in a certain subset of [dynamically possible models, DPMs] of GR"; (n)evertheless, in such instances it is extremely useful to make use of this term, within that subclass of DPMs. Hence, at a practical level, it is legitimate to call such a quantity gravitational stress-energy."

This is an instance of the following principle for realist commitment towards a theoretical, higher-level concept *Q* (cf. Dennett, 1991a; Ladyman & Ross, 2007, esp. Ch. 4) – what Wallace (2012, Ch. 2) dubs "Dennett's Criterion":

(*DC*) Whenever *Q* is definable and explanatorily or predictively useful, it captures a real structure ("real pattern") in the world.

Real patterns are higher-level structures: they are formulated in non-fundamental terms. (Think of molecules and their shapes as treated in chemistry. A satisfactory fundamental account isn't available at present (see Hettema, 2012 for the chemical case). Of course, this doesn't imply that real patterns are "strongly autonomous" (Fodor), i.e. unrelated to the most fundamental level.)

To complete his affirmative answer to (a), Read needs to assume that the quantities conventionally labelled "(formal) gravitational energy", *gravE*$_f$[75], indeed satisfy *(DC)*:

(DC)[gravE$_f$] For certain DPMs, *gravE*$_f$ is definable and explanatorily/predictively useful.

It now follows from *(DC)* that *gravE*$_f$ captures a real pattern in the world ("is real"):

$$(DC)\ \&\ (DC)[\mathrm{gravE}_f] \rightarrow \mathrm{gravE}_f \text{ is real.}$$

Having established the reality of formal gravitational energy, Read's next step is to affirm (b): the real pattern *gravE*$_f$ captures should be identified as *genuine* gravitational energy-stress; it represents gravitational energy-stress also in a substantive*, physical* sense.

Read's rationale encompasses three elements: a general functionalist principle for characterising quantities, a particular functional profile for genuine gravitational energy-stress, and the premise that *gravE*$_f$ exhibits this profile.

Read deploys what he terms a "functionalist" (p. 20) general strategy: "In our view, it is plausible to maintain that in situations such as those in which [the integral conservation law]

[75] For the sake of simplicity, in the remainder of this section *gravE*$_f$ will denote *both* the gravitational pseudo-tensor and its charge.

holds, there exists a quantity in GR that fulfils the functional role of gravitational stress-energy” (pp. 19).

That is, Read adopts the following “functionalism about gravitational energy-stress”:

($FUNC_{gravE}$) For a quantity Q to be (represent, “$\doteq$”) genuine gravitational energy-stress *is* for it to exhibit a certain profile $\mathcal{F}(\mathrm{gravE})$ of functional roles:

$$(\mathcal{F}(\mathrm{gravE}))[Q] \Leftrightarrow Q \doteq \mathrm{gravE}.$$

How to flesh out the functional profile of gravitational energy-stress, $\mathcal{F}(\mathrm{gravE})$? Read determines it to comprise two functional roles:

($\mathcal{F}(\mathrm{gravE})$)
(i) balancing the non-gravitational energy such that the sum is conserved
&
(ii) “(bearing) some relation to the ‘gravitational’ degrees of freedom in the theory in question” (p. 20).

To complete his argument, a final premise is needed - viz. that $gravE_f$ plays the preceding two functional roles:

$(\mathcal{F}(\mathrm{gravE}))[gravE_f]$ $gravE_f$ instantiates the profile ($\mathcal{F}(\mathrm{gravE})$).

By construction, $gravE_f$ obeys a (formal) balance equation. Hence, (i) is satisfied. Likewise, (ii) looks harmless: it’s customary (e.g. Misner, Thorne & Wheeler, 1973, passim) to identify the metric with the gravitational degrees of freedom (the “gravitational field”); $gravE_f$ is directly and solely built from it.

From the conjunction of ($FUNC_{gravE}$) and ($\mathcal{F}(\mathrm{gravE})$) now follows that $gravE_f$ earns the label “gravitational energy”. It represents *genuine* gravitational energy-stress:

$$\left(FUNC_{\mathrm{gravE}}\right) \,\&\, \left(\mathcal{F}(\mathrm{gravE})\right) \,\&\, \left(\mathcal{F}(\mathrm{gravE})\right)\left[\mathrm{gravE}_f\right] \rightarrow \mathrm{gravE}_f \doteq \mathrm{gravE}\,.$$

In summary, Read has thus given a formally valid argument for (*REAL_{LOC}*) & *(REAL_{GLOB})*. Based on the alleged expedience of the gravitational pseudotensor and its associated charge, Read argued for a realist stance towards them. Furthermore, meeting his functional desiderata of gravitational energy, they indeed represent, on his proposal, gravitational energy-stress.

What to make of Read's proposal? Is the appeal to functionalism convincing? Does gravitational energy-stress in GR really satisfy the functional roles, stipulated by Read? Does his proposal overcome the difficulties that undergird Hoefer's eliminativism (§3.3)? To these questions we now turn.

IV.4.2 Objections

In this subsection, I'll evaluate Read's realism about gravitational energy. Apart from Dennett's Criterion *(DC),* and the fact that the formal notions of gravitational energy play the two functional roles stipulated by $\big(\mathcal{F}(\mathrm{gravE})\big)\big[\mathrm{gravE}_f\big]$, I'll question each assumption in his reasoning sketched above.

I'll discuss each premise separately and in increasing order of generality: ($\mathcal{F}(\mathrm{gravE})$), (*FUNC_{gravE}*) and *(DC)[gravE_f].*

IV.4.2.1. Is Read's functional characterisation of gravitational energy-stress adequate?

Consider first Read's functional profile of gravitational energy-stress, i.e. ($\mathcal{F}(\mathrm{gravE})$): are the functional roles of gravitational energy-stress adequately characterised by (i) and (ii)? I dispute that: they are neither jointly sufficient nor necessary.

Two facts cast doubt upon the view that (i) and (ii) are jointly sufficient: the triviality of continuity equations, and ambiguity, respectively.

Firstly, formal continuity equations are too easily procurable (Goldberg, 1958, p. 17). For *any* symmetric quantity $\gamma^{\mu\nu}$, one can always construct a symmetric quantity $\Gamma^{\mu\nu}$ that satisfies continuity equation $\partial_\nu\big(\sqrt{|g|}T^{\mu\nu}+\Gamma^{\mu\nu}\big)=0$ – viz. $\Gamma^{\mu\nu}:=\partial_{\varrho,\sigma}(\gamma^{\mu\nu}\gamma^{\varrho\sigma}-\gamma^{\nu\varrho}\gamma^{\mu\sigma})-\sqrt{|g|}T^{\mu\nu}$. (Recall that that the energy-stress tensor $T^{\mu\nu}$ also depends on the metric, cf. Lehmkuhl, 2011.) If one now chooses for $\gamma^{\mu\nu}$ some arbitrary function, e.g. $\gamma^{\mu\nu}=\sin(R)\,R^{\mu\nu}$, one obtains a quantity that satisfies (i) and (ii). Nonetheless, one would hesitate to ascribe it physical significance as a candidate gravitational energy.

Read may demur at continuity equations thus constructed as they hold irrespective of any field equations (and furthermore that they also depend on the matter degrees of freedom). They are indeed mathematical identities. Read might parry by supplementing (i) with a proviso: the conservation law *not* be a mathematical identity (and not directly depend on the matter degrees of freedom).[76]

This doesn't alleviate the above worry, though: the previous argument can just be rehashed for $\tilde{\Gamma}^{\mu\nu} := \partial_{\varrho,\sigma}(\gamma^{\mu\nu}\gamma^{\varrho\sigma} - \gamma^{\nu\sigma}\gamma^{\mu\sigma}) - \frac{1}{\kappa}\sqrt{|g|}G^{\mu\nu}$. The continuity equation continues to hold – but now in virtue of the Einstein Equations.

Another problem arises from ambiguity. Recall from §2.3: there exist infinitely many pseudotensors satisfying a local continuity equation. All are built solely from the metric. One needn't even restrict oneself to pseudotensors. Nothing in Read's proposal seems to prevent one from introducing e.g. additional flat background metrics, an orthonormal tetrad or a flat connection (Pitts, 2011b for a survey of such options). Ditto quasi-local notions (see e.g. Szabados, 2009).[77] Objects with the functional profile $\mathcal{F}(\mathrm{gravE})$ abound.

Unless their mutual consistency can be established, this proliferation of candidate objects that satisfy $\mathcal{F}(\mathrm{gravE})$ should unsettle Read. (Recall our discussion of (R2) in §3.2.) I therefore conclude: (i)&(ii) is an insufficient characterisation of the functional profile of gravitational energy.

Further scepticism about the functional roles of $\mathcal{F}(\mathrm{gravE})$ is in order. 1. Conserved quantities are contingent on symmetries. Hence, criterion (i) isn't necessary. 2. Criterion (ii) is bedevilled by general fuzziness, as well as equivocation about the gravitational degrees of freedom.

[76] This meshes with common practice in the literature on conservation laws: one distinguishes between "improper" (Hilbert) or "strong" (Bergman) conservation laws on the one hand, and "proper" or "weak" conservation laws on the other (see Brading & Brown 2000; Brading, 2005).
However, whether "proper conservation laws" have physical significance eo ipso is a delicate question (Sus, 2017). As Read's counter-manoeuvre would arguably seek to ensure physical significance, the proviso would have to be formulated carefully.

[77] Quasi-local approaches are plagued by ambiguities of their own, as both Hoefer (2000, p. 196) and Lam (2011, p. 1022) correctly point out.

I'll first argue that (i) imparts a spurious essentiality to a contingent feature of our most familiar spacetime settings.

Underlying Read's stipulation is the intuition that *total* energy should be conserved. This intuition stems from our habituation to classical theories in flat spacetime (cf. Nerlich, 1991). Why expect this to carry over to GR?

The principal motivation stems from the Noether theorems. They establish a general correlation between symmetries of the action and conserved quantities (see e.g. Brading & Brown, 2000). Due to its general covariance, GR's action has infinitely many rigid symmetries (see Bergmann, 1949, 1958; Brown & Brading, 2002; Brading, 2005). The Noether Theorems then guarantee, at least *formally*, infinitely many conservation laws of the pseudotensorial type. To take these formal infinitely many conservation laws seriously, i.e. to regard them as also *physically* meaningful, leads us back to Pitts' proposal. Whether it deserves realism, remains controversial, as we saw.

One source of reservations about the infinitely many conservation laws may derive from GR's general covariance. Because of the latter, they belong to so-called "improper conservation laws" (Hilbert). These arise from Noether's theorems for all theories with local symmetry group that have a global subgroup (see e.g. Bergmann, 1949; Brading & Brown, 2000). Their interpretation and physical significance – as Hilbert's label intimates – is subtle: under certain circumstances, they seem to be (at least, individually[78]) trivial, i.e. *mathematical* identities (see e.g. Brading, 2005; Sus, 2017), and hence devoid of physical content. What those circumstances exactly are, is a question of current dispute (closely related to the empirical significance of symmetries, see e.g. Kosso, 2000; Brading & Brown, 2004; Wallace & Greaves, 2014; Teh, 2015; Murgueitio Ramirez, 2019). On a recent proposal (Barnich & Brandt, 2002; Sus, 2017), GR's improper energy conservation laws can be salvaged from triviality, if the dynamically possible spacetime models considered possess (asymptotic) symmetries. Whether in our world we should take these infinitely many conservation laws seriously, thus depends on whether we should believe that our world instantiates such asymptotic background structure. And indeed, I'll argue below that one should – however, the asymptotic structure is that of a de Sitter space. But that entails two problems. The first is that the

[78] The *totality* of pseudotensorial continuity equations, however, is equivalent to the Einstein field equations (Anderson, 1967, p. 427). Hence, it possesses physical significance (cf. Pitts, 2010, 2016c).

integrals of the pseudotensor-based continuity equations diverge. Thereby, the conserved global/integral charges aren't well-defined. But with the symmetries of de Sitter space, also the motivation for a local/differential conservation law, based on pseudotensors, becomes moot: Using the the associated so-called Killing vectors (see e.g. Read, 2018, sect. 2.4), one can define bona fide (covariant) *matter* energy-stress fluxes that are covariantly conserved – with no (overt) gravitational contributions (Duerr, 2018a, sect. 2).

The connection with Killing vectors can be developed further along a different direction. Unless the spacetime possesses symmetries (to which special coordinates could be adapted (see e.g. Pooley, 2017, sect.), coordinates that would be able to single out pseudotensor-based continuity equations) the pseudotensorial conservation laws thus seem to lack intrinsic meaning. But such spacetime symmetries are contingent: generic spacetimes lack them; *even most* do. Why, therefore, cling to energy conservation as a default? It seems more natural to reverse the familiar explanatory asymmetry: energy *conservation*, *not* its failure, needs explanation – in terms of a spacetime's *special* symmetries (see Carroll, 2010 for a slightly brutal way of putting it; cf. Duerr, 2018a, sect. 2).[79]

Of course, one might resist this whole reasoning by pointing to the *mathematical* fact that, due to general covariance, GR's action has symmetries. But as mentioned before, it's unclear that this, by itself, warrants wider-reaching *physical* conclusions. (Also bear in mind that that most will hesitate to regard an action as more than a merely auxiliary construct − not a physical quantity. Hence, inferences from its properties to properties of physical systems must be handled with care.)

Let's move on to Read's second functional characteristic of gravitational energy, (ii). It can be opposed for two reasons. One is its vagueness: what *exactly* is the relation that should hold between a candidate for gravitational energy-stress and the gravitational field?

A second worry is more subtle: what *are* the gravitational degrees of freedom – the "gravitational field"?[80] Which quantity represents them, e.g. the metric $g_{\mu\nu}$, the connection

[79] One may flesh this out further in terms of Strevens' (2011) notion of difference-making.

[80] An anonymous referee has voiced misgivings that this isn't a serious question for physics: although various definitions are possible for the gravitational potential, the proposed choices don't essentially affect the canonical pseudotensor, understood as the canonical energy-stress associated with the gravitational degrees of freedom. To identify the latter, according to her or him, only the gravitational field (whose role is presumably analogous to that of the electromagnetic field) should be used.

coefficients $\Gamma^{\lambda}_{\mu\nu}$ (Einstein's choice, see Lehmkuhl, 2014), the Riemann tensor (Synge's choice, Synge, 1960), or the deviation from flatness $g_{\mu\nu} - \eta_{\mu\nu}$ (Pooley's choice, Pooley, 2013, fn. 20)?[81] Each choice has some merits in its favour (Lehmkuhl, 2008). Read rightly cautions against any premature a priori preference for one.

Yet, it's not obvious that his second functional role for gravitational energy, (ii), can avoid an a priori choice. The pseudotensors in *(REAL$_{LOC}$)* are the canonical energy-momenta associated with the *metric* as the gravitational field.

Suppose, however, that we identify the *connection coefficients* $\Gamma^{\alpha}_{\beta\gamma}$ as the gravitational field. Then, the associated canonical energy-stress is the Palatini-pseudotensor

$$\vartheta_{\mu}{}^{\nu}[\Gamma] = \frac{\partial \widehat{\mathfrak{L}}}{\partial\left(\partial_{\nu}\Gamma^{\alpha}_{\beta\gamma}\right)} \partial_{\mu}\Gamma^{\alpha}_{\beta\gamma} - \delta^{\nu}_{\mu}\widehat{\mathfrak{L}},$$

where $\widehat{\mathfrak{L}} = \widehat{\mathfrak{L}}\left(g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}, \partial_{\kappa}\Gamma^{\lambda}_{\mu\nu}\right)$ is the (full) Einstein-Hilbert Lagrangian as a functional of the metric, the connection coefficients, and their first derivatives.[82] (Note that it satisfies the continuity equation: $\partial_{\nu}\left(\sqrt{|g|}\left(\vartheta_{\mu}{}^{\nu}[\Gamma] + T^{\nu}_{\mu}\right)\right) = 0$ with the standard energy-stress tensor T^{ν}_{μ}. Again, $\vartheta_{\mu}{}^{\nu}[\Gamma]$ is determined only up to a superpotential term.)
Being 2nd order in the metric compatible with the connection, $\vartheta_{\mu}{}^{\nu}[\Gamma]$ manifestly differs from the Einstein or Landau-Lifshitz pseudotensor, which Hoefer, Lam and Read are considering. What is more, its properties are physically implausible: for instance, it yields divergent integrals for radiating systems (see Murphy, 1990 for details).

One may object: by comparing the Einstein pseudotensor and the Palatini pseudotensor, aren't we comparing apples and oranges? The Palatini pseudotensor $\vartheta_{\mu}{}^{\nu}[\Gamma]$ is based on the *full* Einstein-Hilbert Lagrangian −not (as is the Einstein pseudotensor) on the truncated, "ΓΓ"

I beg to differ. First, arguments from analogy are notoriously defeasible. It's therefore unclear to me that speaking of gravitational potentials is legitimate, let alone illuminating. Secondly, even if we trusted the analogy, how to flesh it out? Which object represents the potential, which the field? For better or worse, we find various options considered in the pertinent physics literature (cf. Lehmkuhl, 2009).

[81] Read (p. 7 p. 20, fn. 35) acknowledges this.

[82] The connection needn't be assumed to be metrically compatible ab initio. A variation of $\widehat{\mathfrak{L}}$ with respect to both the metric and the connection as independent variables enforces metric compatibility. This variational method is called Palatini approach (e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 19.10). The Palatini pseudotensor $\vartheta_{\mu}{}^{\nu}[\Gamma]$ naturally emerges within this approach – hence its label.

Lagrangian $\overline{\mathcal{L}}_{(g)}$. If one determines the corresponding Einstein pseudotensor for the full Einstein-Hilbert Lagrangian, both expressions coincide (Novotný, 1993).

Prima facie, this is satisfying (and a remarkable property of the Einstein-Hilbert Lagrangian!). Nonetheless, it spells a dilemma for Read. One horn is that Read's criterion seems incomplete: it can't decide between the Palatini pseudotensor and the metric-based pseudotensors. If then, in light of the above considerations, one rules out the former, one thereby has to identify the metric as the gravitational field. (Prima facie this isn't implausible: it surely plays a privileged role. For instance, one cannot write down matter coupling to gravity locally using only a connection. One also needs the metric or something equivalent.[83]) But even so, the metric's special status doesn't by itself justify its elevation to the gravitational field – as Read himself admits.[84]

The worry about the right identification of the gravitational field is even more general: why assume that in GR there exists an ambiguously identifiable gravitational field to begin with? It's not implausible that *no* choice for the gravitational field is ultimately unique across different contexts (Rey, 2013).

In short, Read's second functional role, (ii), on pain of incompleteness, cannot remain neutral on the identification of a gravitational field −against his express intentions.

IV.4.2.2. Is a functionalist strategy appropriate for gravitational energy-stress?

Now turn to ($FUNC_{gravE}$): why appeal to functionalism in the *specific* context of gravitational energy in GR? I'll launch two lines of attack against it: first, I'll rebut Read's explicit argument for it; secondly, I'll rehearse the reasons that motivate functionalism in the philosophy of mind, and try to ascertain their analogues.

[83] It's worth recalling that also fermions essentially couple to the connection determined by the metric −not any other connection, cf. Pitts, 2012 for details.

[84] What about the analogy between GR and Yang-Mills Theory? On the one hand, it would indeed *strengthen* the identification of the *connection* as the gravitational field variable. On the other hand, GR *isn't* a Yang-Mills Theory −at least not in the standard sense (see e.g. Aldrovandi & Pereira, 2013, passim). So, from the outset the analogy harbors important subtleties. I therefore side with Read's admonition to caution: what we identify in GR as the gravitational field, requires explicit arguments, and can't be easily read off from the analogy with Yang-Mills Theories.

First, let's examine Read's own argument for a functionalist stance towards gravitational energy. I reject it as unfounded.

What primarily bolsters (*FUNC$_{gravE}$*) for Read is the sterility of its negation: "[…] the alternative to functionalism is to say that 'the structure of certain DPMs of GR is such that it appears that there exists gravitational stress-energy in those models, but really there is no such stress-energy there'; the payoff to be gained from making such a claim is unclear" (p. 20). In particular, he cautions that without (*FUNC$_{gravE}$*), one may be barred from potentially more perspicuous avenues for explaining some gravitational phenomena, e.g. binary systems.

I concur with Read on the infertility of dogmatically boycotting *higher-level explanations* from the outset. Sundry examples from non-gravitational physics attest to that (e.g. Falkenburg, 2015; Knox, 2016; 2017; Knox & Franklin, 2018). Yet, the use of higher-level concepts doesn't per se imply functionalism.[85] The latter is a specific thesis about the meaning and/or the ontological nature of certain quantities (depending on the strain of functionalism, see below). The purported explanatory pay-off of recourse to gravitational energy-stress as a non-fundamental explanans doesn't per se warrant functionalism about gravitational energy-stress.

Moreover, not even the explanatory pay-off of gravitational energy as a higher-level explanans is obvious. Read concedes that appeal to gravitational energy-stress *isn't* necessary: "one could indeed explain all general relativistic phenomena, in any model of the theory, simply using the apparatus used to pick out the [Dynamically Possible Models] of the theory" (p. 20).[86] The existence of two alternative explanations prompts the question: *which* of the two achieves the pay-off that Read extolls? (Contrast this with the case of quasi-particles.

[85] Read's source of inspiration for functionalism in the philosophy of physics is Wallace (2012), whom he quotes (op.cit, p. 58): "Science is interested with interesting structural properties of systems, and does not hesitate at all in studying those properties just because they are instantiated 'in the wrong way'. The general term for this is 'functionalism […].''
This is a gross simplification of functionalism. Wallace's project is primarily concerned with a realist ontology for higher-level/emergent entities. Functionalism is first and foremost the doctrine that what makes an entity to be of a particular type doesn't depend on the entity's composition. Structural realists – such as Wallace (cf. op. cit., pp. 314) – are eo ipso functionalists about all entities, including higher-level ones. Those with different metaphysical penchants, however, can avail themselves of higher-level explanantia without being functionalists about them.
[86] Schutz (2012, p. 7), for instance, writes: "We know today that it is perfectly possible to describe the generation of gravitational waves and their action on a simple detector without once referring to energy; the quadrupole formula for the generation of the waves and the geodesic equation for their action on a simple detector are all one needs […]."

Fundamentally, they are collective excitations in a solid. In some regards, they behave like particles. A bottom-up, statistical mechanical treatment would require utopian computational power: we'd have to solve typically $\sim 10^{23}$ coupled differential equations. The pay-off of the higher-level description is manifest.)

What about binary stars, which Read adduces as an example? The case isn't as clear-cut as Read suggests. GR predicts that two stars revolving each other emit gravitational radiation, and increase their orbital frequency. With marvellous accuracy, this has been confirmed (e.g. Stairs, 2003). In line with Read's claim, the standard account indeed involves gravitational (wave) energy as an explanans (cf. e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 18): the gravitational wave is supposed to carry away the binary system's total (kinetic plus gravitational) energy; as a result, the stars' orbital frequency increases, with the stars spiralling in towards each other.

In a recent detailed analysis, however, Dürr (2018b) – see Ch. II – compares this standard interpretation of the binary stars to the alternative without gravitational (wave) energy which Read adumbrates. The latter is found to trump the former on the four explanatory virtues of parsimony, scope, depth, and unificatory power. At least pro tempore, this diminishes the force of Read's argument, or even shifts the burden of proof upon Read's shoulders.[87]

Two caveats are in order. First, examples *might* eventually be found in favour of Read's claim (e.g. in a similar analysis of instabilities in rotating neutron stars, induced by gravitational radiation, see e.g. Schutz & Ricci, 2010, §6.2).[88] But for the dialectic of the debate to progress,

[87] It's possible to gainsay this conclusion, and still uphold an argumentative asymmetry in favour of Read's position. According to the pragmatic account of explanation, developed by Van Fraassen (1980), what counts as a good explanation is always relative to a particular context. From this angle, *I* seem to make the stronger claim that there is *no* context in which explanatory recourse to gravitational energy-stress pays off. Therefore, it would appear incumbent on *me* to corroborate it. Moreover, the heuristic and didactic benefits seem obvious – not least since appeal to gravitational energy is an almost undisputed practice in the physics literature.
On the one hand, I readily acknowledge *some* merits of explanations with gravitational energy in *certain* contexts. On the other hand, the above counter doesn't sway me for two reasons. First, Read would be ill-advised to be wedded to one *particular* – invariably controversial – account of explanation (cf. e.g. Woodward, 2014). Secondly, the conjunction of the context-relativity of explanations, and Dennett's Criterion entails an outré context-relative ontology. If what counts as a successful explanans depends on the context, and if the role that a successful explanans plays determines what the explanans is, it depends on the context what constitutes a successful explanans. Combine this now with (DC): successful explanantia merit realist commitment. A kaleidoscopic ontology ensues: the world would seem populated with a plethora of motley entities; depending on the context, what exists in the world would vary. This lack of coherence strikes me as unpalatable in an ontology.

[88] The most promising place for such an argument is arguably black hole thermodynamics. I forgo the topic for two reasons. Firstly, the current thesis' ambit is classical GR. I steer clear of any non-classical/quantum aspects. Secondly, the status of black hole thermodynamics is the current topic of dispute, cf. Dougherty & Callender

detailed case-studies of such examples are needed. At the moment, they aren't available. Secondly, some of the persuasiveness of Dürr's (2018ab) arguments depends on whether one shares his GR-exceptionalist creed (see §1). But Read gives no explicit reasons for or against it.

My second line of attack against ($FUNC_{gravE}$) adverts to the motivation for functionalism in the philosophy of mind. I submit, it doesn't carry over to the case at hand.

The functionalism, which Read (via Wallace) imports into the philosophy of physics, stems from the philosophy of mind (see e.g. Van Gulick, 2009; Levin, 2013; Braddon-Mitchell & Jackson, 2007, Part I, II, IV). It's mainly motivated by two difficulties: the non-intersubjectivity of mental states, and the identity theory's failure to account for multi-realisability, respectively.

The first is a general and epistemological point: we can't directly know other people's mental states. They defy inter-subjectivity: a tooth-ache is inherently "private". At best, we can infer mental states indirectly from external indicators (screams, tears, etc.). If thus we want to attribute mental states to other people, prima facie we have to postulate them as entities whose intrinsic nature is elusive. (Mental states might − at best − be accessible introspectively.[89]) It's sound philosophical advice to strive to minimise the gap between our speculations about the world and our knowledge. How then to accommodate for mental states?

A second motivation for functionalism arises from a shortcoming of the preceding identity theory. According to the latter, mental states (or properties) are identical with physical states (or properties). Mental states are multiply realisable: it seems unduly chauvinistic to decree apriori that organisms can't be ascribed the same (or sufficiently similar) mental states, despite neuroanatomical and neurophysiological differences. Why shouldn't, say, Read and an octopus both be able – at least in principle – to experience pain and pleasure? But on the identity theory it remains mysterious, how two intrinsically sufficiently different brain states can be identical with the same mental state.

(2016); Wallace (2017). Hence, it's unclear what inferences to draw from the putative significance of gravitational energy for it.

[89] Dennett (1991b) denies even that.

Both difficulties can be eschewed by characterising mental states not via intrinsic properties of brain states, but via their function: they are individuated by the structural roles they play in a (neuronal) network.

Do these two motivations have counterparts for the case of gravitational energy-stress in GR? Three disanalogies speak against it: its absence in the manifest image, its non-privacy and absence of multi-realisability.

First, on the one hand, gravitational energy-stress – unlike mental states – isn't an empirical phenomenon that *needs* to be accounted for. On the other hand, unlike (say) belief states, even as a theoretical concept, gravitational energy scarcely counts as a robust folk-theoretic notion in our manifest image that an adequate scientific theory in one way or the other *must* save.[90] Read himself acknowledges that it's − at least conceivably − dispensable.

Secondly, being a *physical* quantity, gravitational energy doesn't suffer from the privacy of mental states: nobody is endowed with a privileged introspective access to gravitational energy-stress, opaque to lesser mortals.

A less quirky sense of "privacy" in this context takes its cue from Dennett (1991, cf. Ross, 2000, pp. 161).[91] For him, it's typical of real patterns to become visible only on higher-levels of description. On the fundamental level, one may lose their salience out of sight: one doesn't see the wood for the trees. (This is the sense in which the higher-level explanations, discussed by Knox (2016, 2017) and Franklin & Knox (2017) reveal the salient features, otherwise opaque on the microphysical level.)

[90] Herein lies a key difference to other areas in philosophy of physics where functionalist strategies are deployed. Consider first Everettian quantum mechanics. One of its major challenges is how to recover our manifest image of macro-objects in 3-dimensional space, like crystals and anteaters, from the scientific image of a single, richly structured entity, defined on a higher-dimensional so-called configuration space. The appearance of three-dimensionality is a robust phenomenon that on pain of empirical incoherence arguably needs to be accounted for (e.g. Ney, 2010). To achieve this, Everettians routinely appeal to functionalism (e.g. Wallace, 2012, Ch. 2; Ney, forth.).
Another example are quantum theories of gravity (cf. Le Bihan, forth. for a detailed analysis between functionalism in the philosophy of mind and philosophy of quantum gravity) devoid of familiar (e.g. smooth) spatiotemporal structure (Lam & Wüthrich, 2018). Given that the latter is a robust phenomenon, one must arguably be able to give a story of how to recover it from our "a-spatiotemporal" scientific image.
Notice also that, by contradistinction, Knox's "inertial frame functionalism" clearly doesn't aim at recovering the manifest image. In this regard, then, Wallace's and Lam & Wüthrich's projects are closer to functionalism in the philosophy of mind than are Knox's or Read's respective uses.

[91] I thank James Read (Oxford) for alerting me to this possibility.

Is gravitational energy "private" in this sense? Can it only be properly understood on the coarse-grained, higher-level which Read's functionalist perspective envisions? That, too, I impugn. Formal notions of gravitational energy aren't *higher*-level concepts in the relevant sense: they are non-fundamental in that they are only definable in certain subclass of models. Again, the motivation from "privacy" founders.

Thirdly, multi-realisability has no obvious counterpart. Recall that it's an inter-level relationship: it links higher-level and lower-level (more fundamental) entities. ($FUNC_{gravE}$) presupposes that the functional profile of gravitational energy is supplied from gravitational theories other than GR.

The most straightforward such "reference theory" is Newtonian Gravity. GR reduces to it in the weak gravity limit.[92] Hence, the functional role would be fixed by GR itself in a particular regime. (One may already ponder: isn't it ad-hoc to accord an *ontological* privilege to this particular regime? The more modest goal of identifying rough-and-ready functional *counterparts* of quantities in antecedent theories is, of course, harmless. See below.) Suppose now that in another regime, GR exhibits some structural similarity to the weak-field regime. This similarity doesn't constitute an *inter*-level relationship of the kind required for multi-realisability. It doesn't link a fundamental and a less fundamental level of description. Rather it's an *intra*-level relationship. The same applies to different reference theories, say massive graviton gravity.[93] Both GR and it vie for providing the best description of the same domain. They operate on the *same* ontological level. Again, we aren't dealing with multi-realisability.

It's terminological confusion to say that GR "instantiates" or "realises" some quantity, defined in massive graviton gravity. Of course, one could meaningfully ask: what structures of a GR spacetime are the (rough) *analogues* or *counterparts* of some quantity in massive graviton

[92] Herein lies another, albeit arguably peripheral, difference to the philosophy of mind: it's not even clear how best to *conceptualise* a potential reduction of the mental to the physical – let alone whether it can successfully be carried out (cf. e.g. Beckermann, 2008, Ch. 8,9).
[93] Massive spin-2 graviton theory (e.g. Hinterbichler, 2012; de Rham, 2014) happens to be empirically adequate for suitable field masses (Pitts & Schieve, 2007; Pitts, 2011a; 2016).

gravity?[94] But gleaning structural similarities is ontologically much less ambitious than Read's realism.[95]

In short: The two main motivations for functionalism in the philosophy of mind −non-intersubjectivity of mental states and multi-realisability− lapse for gravitational energy-stress. This corrodes any tangible motivation for ($FUNC_{gravE}$).

This conclusion calls for qualification. Plausibly, the above motivations are (individually) sufficient conditions for the application of functionalism. I *don't* claim that they are necessary. But to my knowledge, there aren't any *other* motivations for functionalism in the literature. Hence, it seems not unfair to request of Read a justification of his functionalist strategy, should it be motivated "non-standardly".

IV.4.2.3. Is Dennett's Criterion really satisfied?

Let's eventually revert to Read's reality criterion, (DC). To decide whether the formal concepts of gravitational energy, gravE_f, capture real structures, Read employs Dennett's reality criterion (DC): *if* a higher-level quantity is well-defined *and* explanatorily/predictively useful, it merits realist commitment. Are the antecedent conditions really satisfied?

Let's hark back to the main finding of our more careful exegesis of Hoefer's first objection, *(H1),* in §3: realism about local and global pseudotensorial gravitational energy-stress, *($REAL_{LOC}$)* and *($REAL_{GLOB}$)*, is obstructed both by the pseudotensor's ambiguity/non-uniqueness

[94] "Analogue gravity" illustrates this. Some non-gravitational systems, e.g. Bose-Einstein condensates or sound waves in relativistic fluids, can *simulate* certain features of general-relativistic gravity (see e.g. Visser, Barceló & Liberati, 2011). Rather than instances of multiple realisations of (general-relativistic) gravity, they merely provide insights by exploiting analogical structures (cf. Dardashti, Thébault & Winsberg, 2015 for a philosophical analysis).

[95] This is how I classify the "spacetime functionalism", promulgated by Knox (2013, ms, 2017): it uses the functional role of GR's spacetime to identify the counterparts of (i.a.) GR's inertial structure in other spacetime theories. This is an application of Lewis' (1970, 1972) proposal of functional definitions. Via the latter, one determines correspondences between theoretical terms of two theories rather than (Nagelian) reductions via bridge laws.

In other words: Knox's "spacetime functionalism", to my mind, is a form of "commonsense functionalism", which uses the functional role as a reference-fixing device (cf. Braddon-Mitchell & Jackson,2007, Ch. 3,15). I regard it as the strength of Knox's position that it's not inherently committed to anything ontologically more ambitious (e.g. regarding this functional role as constituting the *essence* of spacetime).

In particular, my reading of Knox's spacetime functionalism as a "commensense functionalism" is congenial to Baker (2019), who argues that spacetime is a cluster concept: spacetime structure in various theories plays many roles, with no single role being sufficient or necessary for spacetime simpliciter. This is exactly what to expect of correspondences between terms belonging to distinct theories: usually, there are no unique, one-to-one correspondences.

and vicious coordinate-dependence (unless Read's position collapses onto Pitts' – a position for which no independent arguments have been given). Read's responses to Hoefer were seen to be either ineffective or incomplete. His functional approach discussed added nothing relevant as regards these problems: the first antecedent condition of *(DC)* isn't satisfied: gravitational energy-stress isn't well-defined (except for Pitts' object).

What about the other condition – explanatory utility? Read still owes us an argument, or full-fledged example, for why gravitational energy-stress is a powerful explanans.[96] To my mind, this can only be satisfactorily gauged through detailed case studies (e.g. of energy extraction processes in Black Holes, see e.g. Geroch, 1973). Nonetheless, a strong argument for eliminativism can already be made, turning on a wide range of astrophysical and cosmological phenomena.

Recall that Read's *(REAL$_{GLOB}$)* hinges on realism about asymptotic flatness. In §3.3, I suggested that the disagreement between Hoefer and Read over the acceptability of asymptotic flatness best be understood as a disagreement between different classifications: whereas for Read asymptotic flatness is a good approximation, for Hoefer it's an idle posit in an idealisation. The bone of contention is therefore: do the salient features of empirically confirmed asymptotically flat models successfully refer? Primarily in light of contemporary cosmology, I contend, they don't.

On the one hand, many spacetimes utilised for modelling the exterior of stationary astrophysical objects are indeed asymptotically flat. Apart from the Schwarzschild metric, the the Kerr-Newmann solution for the exterior of a rotating, charged black hole is a case in point (cf. Reiris, 2014 for a proof of a large class of spacetimes). But unfortunately, no interior solution for the (uncharged) Kerr metric is known whose source is a perfect fluid – the simplest model for a star.

This may merely be deplorable. But more generally, Christodolou and Klainerman (1993, p. 10) warn: "[…] it remains questionable whether there exists any nontrivial (non-stationary) solution of the field equations that satisfies the Penrose requirements [i.e. the geometric conditions encoding asymptotic flatness]. Indeed, his regularity assumptions translate into fall-off conditions of the curvature that may be too stringent and thus may fail to be satisfied

[96] Note that Dennett (1989, Ch. 2, 3; 1991a; 2009) accentuates the importance of the *immense* gain in explanatory power – nigh-universal scope and practical ineliminability – for the bona fide applications of his criterion.

by any solution that would allow gravitational waves. Moreover, the picture given by the conformal compactification fails to address the crucial issue of the relationship between the conditions in the past and in the behaviour in the future."

The only known non-stationary, asymptotically flat solutions (e.g. within the Robinson-Trautmann class of metrics describing expanding gravitational waves) are marred by singularities. This threatens their physicality.

There are two responses to this. One is that singularities may not be as calamitous as orthodoxy (e.g. Earman, 1995, p.12) has it (Curiel & Bokulich, 2009, sect. 2; Lehmkuhl, 2016). Another reaction points to approximate solutions based on perturbative methods. Via them one can determine the spacetime of, say, an in-spiralling compact binary system, yielding a spacetime that is non-stationary and asymptotically flat.

This leads us to the major objection to asymptotic flatness as an approximation − cosmology. Prior to that, though, let's briefly dwell on the perturbative approximation schemes featuring in the treatment of binary systems. In a nutshell (see e.g. Maggiore, 2007, Ch. 5; Poisson & Will, 2014 for details), in the astrophysical system's neighbourhood, one employs the so-called Post-Newtonian approximation scheme −an expansion in powers of a small parameter $(1/c^2)$− to determine the system's near field. But this expansion in the near-zone expansion is a singular perturbation theory: for distances tending to infinity, higher-order terms blow up; the Post-Newtonian scheme isn't uniformly valid for all distances. In particular, it cannot incorporate the no-incoming radiation boundary conditions, apt for gravitationally radiating objects. One therefore adopts a different approximation scheme for the so-called "far-field zone". In the intermediate region, both expansions are then smoothly glued together ("matched asymptotic expansion"). Which boundary conditions to impose for the far-field zone? A standard choice is asymptotic flatness.

Here lies the principal reason for classifying asymptotic flatness as an *idealisation*: according to today's best cosmological model, we live in an FLRW universe with a positive cosmological constant Λ. It leads to infinite (albeit ever slower) expansion in our universe's long-term future: our universe is asymptotically deSitter; it's *not* asymptotically flat (see e.g. Carroll, 2003; Rubin & Hayden, 2016 for details).

Already for the exterior of the simplest, i.e. spherically symmetric star model, immersed in a deSitter spacetime, asymptotic flatness breaks down. Does this vitiate all −well-confirmed!− calculations based on an asymptotically flat far-field? Luckily − no: Far from the source, but still much closer than cosmological scales, spacetime *is* approximately flat – for all practical purposes. So, the usual techniques apply – as long as one doesn't venture too "far out" in space and time (Ashtekar, Bonga & Kesevan, 2016; Bonga & Hazboun, 2017).

Asymptotic flatness is therefore an *idealised* extrapolation of the ambient spacetime at a particular phase of a star's life: one ignores its future beyond a certain point, prescinding from the star's cosmic embedding. The referents of asymptotically flat spacetimes are therefore ahistorical *fictional* objects. The practising physicist uses them as convenient surrogates for the real target objects, e.g. a pulsar, a galaxy, etc., because they share with the latter the relevant structural features *up to cosmological scales.* (It's this omission of actual history that physicists mean, when taking asymptotic flatness to characterise isolated systems. An object in an asymptotically flat spacetime is dynamically isolated in the sense that it quiesces into a stationary state.[97]) Asymptotically flat spacetimes thus are idealisations. Even when successful, they describe surrogate systems, distinct (with respect to their past or future evolution) from remotely physical ones.

More importantly, the working posits of successful asymptotically flat models aren't their fall-off behaviour at infinity. Rather, they are the right fall-off behaviour *up to cosmological scales*: all empirical content is garnered from the properties of a *finite* patch of an asymptotically flat spacetime. But it's, of course, the behaviour *at infinity* that is salient of asymptotic flatness. Asymptotic flatness is therefore an idle posit. Recourse to *(DC)* is thus blocked.

In short: Gravitational energy in Read's proposal contravenes both conditions of Dennett's Criterion. Owing to its coordinate-dependence and ambiguity, local and global gravitational energy is ill-defined (unless Read's position collapses onto Pitts', for which then he should argue explicitly). Moreover, asymptotic flatness is an idle posit. Hence, it doesn't yield the explanatory mileage that a realist would urge.

I conclude that Read's argument for a realism about pseudotensor-based global and local gravitational energy fails. In consequence, vis-à-vis Read's proposal, Hoefer's alternative

[97] I propose that Nerlich's main argument (2013, pp. 159) should be understood (more charitably) along these lines: asymptotic flatness imposes a stationary long-term future – contrary to our best cosmological knowledge.

seems preferable. It cuts the Gordian knot: we should indeed be eliminativists about gravitational energy, and recognise that in GR, energy just ceases to be conserved as a default (see Schrödinger, 1950, p. 105 for a "singularly striking example", cf. Misner, Thorne & Wheeler, 1974, §19.4).

IV.5. Outlook

While I argued that Read's proposal should be rejected, his general functionalist approach to gravitational energy can be salvaged, and prove fecund in two slightly different contexts. For that, though, we must be clear on what it is – a scheme that allows us in a principled manner to

1. assess when a (cautious) realist stance towards certain non-fundamental quantities is apposite – via Dennett's Criterion;
2. identify those as counterparts of Newtonian gravitational energy in other theories –via Lewisian functional definitions.

One such promising context concerns non-pseudotensorial approaches to global gravitational energy-stress; the other concerns a research programme inaugurated recently by Ashtekar and collaborators.

Whilst Read doesn't mention them, three other candidates for global gravitational energy lend themselves to his agenda (as I believe, it ought to be understood): the Komar mass, the Bondi-Sachs mass, and the ADM mass. Being non-pseudotensor-based, they circumvent the two main defects of pseudotensors discussed above. To gauge the prospects, I'll comment on each. (I'll skip the technical details. For them, I refer to Wald, 1984, Ch. 11.2; Poisson, 2004, Ch. 4.3; Jaramillo & Gourgoulhon, Ch. 3, and references therein.)

Start with the Komar mass. I submit, it violates either the first or the second antecedent conditions of Dennett's Criterion.

For *stationary* (and asymptotically flat) spacetimes, it furnishes a well-defined, coordinate-independent notion of global gravitational energy. But this augurs only a Pyrrhic victory for Read. The casualty is physical significance: stationarity precludes astrophysical processes like stellar evolution, gravitational or electromagnetic radiation. Realism about the Komar mass thus is Pickwickian: its limited applicability is at variance with any demand for explanatory utility, i.e. the second condition of (DC).

The only spacetimes capable of describing in any sense realistic systems, and hence the only ones capable of empirical confirmation, are of course *non*-stationary. But for non-stationary, asymptotically flat spacetimes, the Komar energy requires a gauge-fixing in the following sense: for the integral to be well-defined, a coordinate condition needs to be imposed on the representative of the equivalence class of so-called Bondi-Metzner-Sachs time translations. (Loosely speaking, the latter encode translations at infinity, see e.g. Wald, 1985, pp. 283 for details.) This gauge-fixing amounts to privileging a certain subclass of time-translations. (NB: The problem *isn't* the imposition of a coordinate condition per se. Rather it's the fact that thereby one singles out time-translations along directions that aren't intrinsically distinguished.) This looks like a drastic, if not ad-hoc restriction of the concept of energy. It thus seems that for empirically relevant contexts, the Komar mass violates the demand for well-definedness, i.e. the first condition of (DC).

A second approach to gravitational energy is the Bondi-Sachs mass. For asymptotically flat spacetimes, it's defined as an integral at "null infinity". Roughly speaking, that is: One evaluates the solution-valued Hamiltonian of GR in the limit surface at infinity along the light cone. (Equivalently, one can conceive of the Bondi-Sachs quantities as Noether charges, associated with the symmetries of asymptotically flat spacetimes at null infinity.) The Bondi mass captures the energy that electromagnetic or gravitational radiation carries off to infinity. In the presence of an outward energy flux, the Bondi-Sachs mass decreases. But it always remains non-zero. It's also bounded from above by the third candidate for gravitational energy in asymptotically flat spacetimes − the ADM mass.

In contrast to the Bondi-Sachs mass, it's defined at "spatial infinity": one evaluates the solution-valued GR Hamiltonian in the limit of *spacelike* hypersurfaces stretching to infinity. The ADM mass is a suitable candidate for *total* energy-momentum of spacetime. By construction, it's conserved. A celebrated result of mathematical physics is that the ADM mass can be shown to be positive (for matter satisfying certain energy conditions). Furthermore, under suitable conditions, it initially coincides with the Bondi-Sachs mass. Accordingly, the latter can be interpreted as the residual ADM energy after gravitational and electromagnetic wave energy has been extracted from the system.

What to make of the Bondi-Sachs and ADM mass in the present context? The reflections on the status of asymptotic flatness as an idealisation in §4.3.2 curtail rash hopes: With asymptotic flatness as their prerequisite (Jaramillo & Gourgoulhon, 2010) both the Bondi-Sachs and ADM mass don't seem to merit realist commitment.

In short, Read's functionalism about gravitational energy doesn't fare significantly better for the three standard non-tensorial notions of global gravitational energy. The Komar mass is either ill-defined or deficient in explanatory power. Both the (standard) Bondi-Sachs and the ADM mass presuppose asymptotic flatness. With the latter being an idle posit of an idealisation, neither seems to merit realist commitment.

Another context, however, deserves greater attention. In it, Read's proposal (understood as sketched above, and with the suitable amendments with respect to the characterisation of the functional profile of gravitational energy) may prove valuable: the framework for asymptotic structure of spacetimes *with* a cosmological constant, $\Lambda > 0$, recently developed in a series of papers by Ashtekar, Bonga & Kesavan (2015abc). It promises to circumvent some of the problems diagnosed for Read's approach. In particular, given that our universe is arguably asymptotically deSitter, the asymptotic structure on which Ashtekar et al.'s framework relies may well count as a working posit of an approximation. It remains to be seen whether the symmetries of deSitter space allow for a satisfactory formal definition of gravitational energy – and what the functional roles are that it plays. But supposing that gravitational energy does admit of a well-defined expression in this context, prima facie it's a counterpart of Newtonian gravitational energy, about which we *should* be higher-level realists.

This chapter:

We identified, in a manner more circumspect than in the preceding chapters, the profound challenges that any realist about local gravitational energy in General Relativity faces, even as a non-fundamental quantity: inter alia, it doesn't form a geometric object. Provided that global notions are formally definable, I argued that only their explanatory utility would warrant a realist stance towards them – a task for future research.

The next chapter:

How does the situation look in other gravitational theories? We'll next inspect the status of gravitational energy in the various (geometric and non-geometric) formulations of Newtonian Gravity.

V. Gravitational Energy in Newtonian Theories

Abstract:

This chapter investigates the status of gravitational energy in Newtonian Gravity (NG), developing upon recent work by Dewar and Weatherall. The latter suggest that gravitational energy is a gauge quantity. This is potentially misleading: its gauge status crucially depends on the spacetime setting one adopts. In line with Møller-Nielsen's plea for a motivational approach to symmetries, I supplement Dewar and Weatherall's work by discussing gravitational energy-stress in Newtonian spacetime, Galilean spacetime, Maxwell-Huygens spacetime, and Newton-Cartan Theory (NCT). Although I ultimately concur with Dewar and Weatherall that the notion of gravitational energy is problematic in NCT, the analysis goes beyond their work. The absence of an explicit definition of gravitational energy-stress in NCT somewhat detracts from the force of Dewar and Weatherall's argument. I fill this gap by examining the supposed gauge status of prima facie plausible candidates – NCT analogues of gravitational energy-stress pseudotensors, the Komar mass, and the Bel-Robinson tensor. The chapter further strengthens Dewar and Weatherall's results. In addition, it sheds more light upon the subtle link between sufficiently rich inertial structure and the definability of gravitational energy in NG.

Key words: Gravitational energy, Newtonian Gravity, Newton-Cartan Theory, pseudotensors

V.1. Introduction

Energy is a pivotal concept in all of physics. Ubiquitous − not least via the 1st Law of Thermodynamics − it has even been argued (by e.g. Bunge, 2000) to be the salient metaphysical property of matter. It's therefore enticing to inquire into the status of the energy associated with the gravitational degrees of freedom in Newtonian Gravity (NG): does NG admit of a meaningful definition of gravitational energy?

The question is of interest for at least three reasons. First, one would like to learn what makes gravity special, vis-à-vis other physical entities – perhaps already at the pre-general-relativistic level. Second, given that NG is an action-at-a-distance theory, one may wonder: does this fact impinge upon the definition of a local notion of gravitational energy? With General Relativity (GR), as a local field theory, in mind, one may ask: to what extent is NG free from the conceptual and interpretative difficulties of quasi-local notions of gravitational energy in GR, (cf. e.g. Szabados, 2009)? Third, it's well known (e.g. Misner, Thorne & Wheeler, 1973, Ch. 12) that NG can be cast in a purely geometrical form, analogous to GR: in it, gravitational phenomena are re-conceptualised as manifestations of a non-flat spacetime geometry. This geometrisation seems to be linked to GR's notorious conceptual difficulties with respect to finding a meaningful notion of gravitational energy (cf. Norton, 2014). Echoing Bunge's suggestion, one might think, these difficulties in defining gravitational energy in GR intimate that gravity isn't a *matter* field, i.e. of the same type as the electromagnetic one (cf., for instance, Sotiriou, Faraoni & Liberati, 2008, sect. 5.2). Studying NG in its geometrised form in closer detail thus promises to help us to better understand such conceptual difficulties – especially given that gravitational energy in NG's un-geometrised form is well-understood and unproblematic. Or so it appears.

In a recent paper, Dewar and Weatherall (2018) have challenged this. They assert that gravitational energy in Newtonian gravitational theories fails to be well-defined: that it's gauge-variant. The current chapter responds to this claim. Notwithstanding our agreement with Dewar and Weatherall's overall conclusion, we feel that their reasoning leaves a few more things to be said – both formally and in substance. In particular, they don't attend to the question whether the spacetime setting makes a difference to the status of gravitational energy in NG. Furthermore, as Dewar and Weatherall (op.cit, p. 13) expressly acknowledge, their reasoning is predicated on a particular view of what counts as a gauge transformation. According to this view, Newton-Cartan Theory *just is* a gauge-invariant reformulation of NG. We explore the consequences of a different understanding of gauge – one which, we contend, is closer to orthodoxy in the philosophy of physics literature (for better or worse).

We proceed as follows. Section 2 clarifies some preliminaries about gauge-invariance and models (§2.1), and then reviews three non-geometrised classical spacetime settings for Newtonian Gravity: Newtonian spacetime (§.2.2), Galilean spacetime (§2.3), and Maxwell-Huygens spacetime (§2.4); for each, the status of gravitational energy is assessed in detail. Section 3 focuses on Newton-Cartan Theory (NCT). §3.1 outlines NCT's basics. In §3.2, we explain why Dewar and Weatherall's objections against gravitational energy in NCT are specious. In §3.3, we try to fill the gap in their reasoning. §4 discusses the results achieved, and their relation to Dewar and Weatherall's own conclusions.

V.2: Gravitational energy in classical spacetimes

Dewar and Weatherall raise a deep question about the status of gravitational energy in Newtonian Gravity (NG): is it a well-defined physical quantity? An answer isn't straightforward: depending on which space-time setting one adopts, *different* variants of NG ensue (Friedman, 1983; Maudlin, 2012; Weatherall, 2016a). As a result, the status of gravitational energy shouldn't be expected to be the same ab initio. After preliminaries about gauge-invariance (§2.1), this section reviews NG within the three non-geometrised classical space-time settings: Newtonian space-time (§2.2), Galilean space-time (§2.3) and Maxwell-Huygens space-time (§2.4), respectively.

V.2.1. Models and gauge-invariance

In this section, we clarify two concepts germane to the subsequent analysis: the classes of models relevant for us, and the notion of gauge equivalence.

Consider a given classical (non-quantum) theory T. Its associated models consists of *n*-tuples of geometrical objects, defined on a space-time manifold $\mathcal{M}$ (for details, see Trautman, 1965;

Anderson, 1967). Some of these represent matter variables, e.g. particle positions or field configurations. The remaining objects represent space-time structure, encoding e.g. chronogeometric or inertial structure.[98]

The various space-time settings correspond to different choices for the space-time structure. Each choice *ought* to conform to Earman's adequacy condition: the spacetime symmetries should match the dynamical symmetries (Earman, 1989, pp. 45). That is: The maximal group of diffeomorphisms under which the dynamical matter variables are invariant should coincide with those under which the spacetime structures are invariant.[99] These matter variables are introduced as follows.

A theory's most general class of models is called its "kinematically possible models" (KPMs). According to Curiel (2016), they specify two things.

1. A specification of the theory's ontology (in the sense of Quine, 1951): The KPMs individuate possible *kinds* of objects to which the theory T is applicable − e.g. a viscous fluid or an electromagnetic field;
2. a specification of the theory's ideology (in the sense of Quine, ibid.): The KPMs enumerate (without determining) the degrees of freedom that form the complete state of possible objects of that kind at a point in time.

The laws which, according to T, relate the entities in the KPMs are given by dynamical equations. In particular, these laws fix their law-like interrelations (e.g. diachronic evolution). They pick out of the KPMs the *dynamically* possible models (DPMs). (One may conceive of the KPMs as representing *T*'s *metaphysically* possible worlds, say, of viscous fluids or electromagnetic fields. The DPMs describe *nomologically* possible worlds; in them, the laws of nature prescribed by *T* hold. Models representing *particular* worlds − e.g. the actual one we inhabit− are obtained, if one further restricts the DPMs by boundary (or initial) conditions.)

Occasionally, one may wish to interpret a multiplicity of DPMs as representing the *same* world. This constitutes a gauge redundancy. On a mainstream view (which we won't call into question

[98] We needn't embroil ourselves in the question of whether such a matter/space-time dichotomy can be upheld categorically (e.g. Maudlin, 1988; Hoefer, 1996; Rynasiewicz, 1996; Rovelli, 1997; Brown, 2005; Rey, 2013; Knox, 2017; Martens & Lehmkuhl, ms).

[99] The transformation isn't to be applied to fixed fields in the theory. Here, a field is called 'fixed', if it is identically the same in every kinematically possible model (to be defined further below), cf. Belot, 2007, p. 197, fn 137.

here[100]), for an object to represent a meaningful physical quantity, it must be gauge-independent. Else, it lacks intelligible identity conditions: the properties of an object violating gauge-independence are unclear.

The present chapter isn't concerned with discussing the criteria of *when* to physically identify two models (cf. e.g. Greaves & Wallace, 2014; Dewar, 2015, 2017; Møller-Nielsen, 2015; Martens & Read, ms) – nor with the pondering on the question when to identify two *theories* (cf., for instance, Read & Møller-Nielsen, 2018). While we won't critically discuss different positions on these matters, we'll nonetheless adopt a cautious stance. Regarding the identification of two models, we'll follow Møller-Nielsen's "motivational approach" (see below). Regarding the identity of gravitational theories, Dewar and Weatherall adopt the latter's own criterion (Weatherall, 2016b): two empirically equivalent theories are merely reformulations of the same theory, if they are categorically equivalent to each other. This stands in opposition to a more traditional view of theory equivalence, such as Glymour's (1970; 1977). When discussing the variants of NG in the various space-time settings, we'll side with the received view: contrary to Dewar and Weatherall, we'll treat them as *different* theories. To our mind, the absence of any consensus on such conundrums about theory equivalence (cf. Dasgupta, 2018; Ismael, 2018) commends cautious conservatism. A given formalism can be interpreted in multiple ways. Whether it's to be regarded as equivalent to a theory couched in a different formalism depends on this interpretation.

Dewar and Weatherall aver that gravitational energy density in NG lacks gauge-independence. This claim deserves scrutiny in each of the canonical non-geometric space-time settings. The remainder of the section will tackle this.

V.2.2 Newtonian spacetime

Let's first consider NG set in Newtonian space-time (NST), NG_{NST}. Its KPMs consist of the 7-tuple

$$\langle \mathcal{M}, t_{ab}, h^{ab}, \sigma^a, \nabla, \varphi, \varrho \rangle.$$

[100] Cf., however, Rovelli (2014) for a discussion and a contrary position.

Here, $\mathcal{M}$ denotes the smooth, 4-dimensional differentiable manifold of events in space-time. t_{ab} and h^{ab} are smooth, symmetric tensor fields on $\mathcal{M}$, of signature $(1,0,0,0)$ and $(0,1,1,1)$, respectively. That is (see Malament, 2012, pp. 49): $\forall p \in \mathcal{M}: \exists \left(\xi^a_{(b)}\right)_{b=0,\dots,3} \in T\mathcal{M}$ such that $t_{ab}\xi^a_{(c)}\xi^b_{(d)} = \delta_{c,d}\delta_{c,0}$.

Analoguously, for the spatial metric, $\forall p \in \mathcal{M}: \exists \left(\sigma^{(a)}_b\right)_{a=0,\dots,3} \in T^*\mathcal{M}$ such that $h^{ab}\sigma^{(c)}_a\sigma^{(d)}_b = \delta^{c,d}(\delta^{c,1} + \delta^{c,2} + \delta^{c,3})$. The two fields represent a temporal and a spatial "metric", respectively. Due to their degeneracy, they aren't metrics proper. In particular, whilst being able to raise indices with h^{ab}, we can't lower them with an inverse metric. For these two metrics, the following three conditions hold ("orthogonality", and "temporal" and "spatial metric compatibility", respectively):

$$h^{ab}t_{bc} = 0$$

$$\nabla_c h^{ab} = 0 \;\&\; \nabla_c t_{ab} = 0.$$

Given a vector field ξ^a, its temporal length is defined via $(t_{bc}\xi^b\xi^c)^{1/2}$. The vector field is called time-like or space-like, if its temporal length is positive or zero, respectively. (For the analogous spatial "metric" we refer to Malament, 2012, pp. 252. The details subsequently play no important role.)

The vector field σ^a is time-like (in the sense that $t_{ab}\sigma^a \neq 0$). Its integral curves represent the persisting points of absolute space. It grounds a standard of absolute rest/motion.

∇ is a flat derivative operator on $\mathcal{M}$:

$$\mathrm{R}^a_{bcd} = 0.$$ [101]

It supplies the geodesic/inertial structure -loosely speaking: a standard of straightness- in terms of which inertial motion is defined.[102]

[101] Recall that this means that the concatenation of parallel transporting a vector ξ^a commutes:

$$\nabla_{[a}\nabla_{b]}\xi^c \equiv R^c_{dab}\xi^d = 0.$$

[102] It deserves to be underlined that the role of inertial structure isn't *exhausted* by explaining (or grounding) inertial/force-free motion (see e.g. Pooley, 2012, sect. 5.2).

The gravitational potential and its source, the mass density, are represented by the smooth scalar fields φ and ϱ. (For simplicity, we'll ignore in the following trivial gauge transformations of the potential, $\varphi \mapsto \varphi + \varphi_0$, for constant φ_0.[103])

Throughout, we'll assume that classical space-times are temporally orientable. That is: There exists a continuous, globally defined covector field t_a such that $t_{ab} = t_a t_b$. A time-like vector ξ^a is future-directed, if $\xi^a t_a > 0$. Otherwise, it's past-directed. In conjunction with the orthogonality and metric compatibility conditions, orientability allows us to slice up a spacetime into simultaneity hypersurfaces (see Malament, 2012, pp. 217).

In DPMs of NG_{NST}, the gravitational potential φ obeys the Newton-Poisson Equation,

$$h^{ab}\nabla_a\nabla_b\varphi = 4\pi\varrho.$$

Consider now Galilean (static and kinematic) shifts:[104]

$$\Gamma: \begin{cases} t \\ x^i \end{cases} \mapsto \begin{cases} t + t_0 \\ x_0 + R^i_j x^j + v^i t \end{cases}.$$

They comprise uniform time (t_0) and space translations (x_0), time-independent spatial rotations (R^i_j), and constant velocity boosts (tv^i) in absolute space. In geometric terms this translates into linear transformations of the type $\sigma^a \mapsto S^a_b\sigma^b + \sigma^a_0$ for a constant vector field σ^a_0 and a constant orthogonal matrix S^a_b with $\det(S^a_b) = 1$ (Earman, 1989, Ch. 2).

In NG_{NST}, kinematic shifts reflect meaningful differences. (Throughout, we'll adopt the position known as sophisticated substantivalism, see e.g. Pooley, 2012, pp. 59. It denies that static shifts – uniform time and space translations – correspond to physically distinct possibilities.) They describe distinct, velocity-boosted worlds. A material reference body in kinematically shifted models moves at different velocities v^i relative to the persisting points of absolute space.

[103] Strictly speaking, in order for this shift to be regard as trivial, one must embrace a form of anti-quidditism about properties. See Martens & Read, ms for details.
[104] This terminology follows Huggett (1999).

Already Newton[105] himself, in his Corollary V, acknowledged the symmetry of models of NG_{NST} under Galilean shifts: its laws remain invariant under them; with respect to the laws, Galilei-shifted models are indistinguishable.

Dynamical shifts generalise kinematic ones. They allow for an arbitrary time-dependent translation $\vec{d}(t)$, concomitant with a re-scaling of the potential:

$$\Delta: \begin{cases} \vec{x} \\ \varphi \end{cases} \mapsto \begin{cases} \vec{x} + \vec{d}(t) \\ \varphi - \ddot{\vec{d}} \cdot \vec{x} + f(t). \end{cases}$$

In a dynamical shift, one subjects the system to a uniform acceleration, $\ddot{\vec{d}}(t)$, and adds a force that remains constant on simultaneity surfaces. (The question of how to translate this into the coordinate-free language of differential geometry needn't distract us here; we'll return to it in §3.1.) A fortiori, dynamical shifts mediate meaningful differences: they represent universes in which some material reference body moves at different (uniform) accelerations with respect to the persisting points of absolute space.

Two models of NG_{NST}, related via dynamical shifts, thus also represent distinct worlds.[106] (We'll see presently that they are nonetheless *observationally* indiscernible.)

According to Dewar and Weatherall, dynamical shifts threaten the gauge-invariance of gravitational energy. To see how, let's first introduce the energy density of the gravitational potential φ as the Noether-current associated with time-translation invariance of the NG Lagrangian,

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[105] "The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion" (Newton, 1729).

[106] We'll set aside here the question of whether dynamically shifted models still constitute solutions of NG_{NST}. Potential doubts might arise from the fact that the "sourceless sources", driving such shifts, are inimical to the Newtonian framework.
At least as it stands, this argument doesn't sway us. First, the historical Newtonian framework has no direct bearing on the systematic question at hand. Secondly, and more importantly, to assess the question from a systematic angle, one must spell out what one means by, and what is included in the "Newtonian framework". An explicit argument must then be given why "sourceless sources" are indeed prohibited within it. (For instance, it's not obvious that the Newton's Third Law is applicable: it refers only to forces – and one may deny that dynamical shifts constitute forces proper. With forces being arguably causes, we have hereby touched on a subtle question in the metaphysics of causation within Newtonian physics.)
Given the lack of a robust consensus on the details of the metaphysical framework appropriate to Newtonian physics – and the ineluctable disputes concomitant with metaphysical frameworks quite general – we deem it prudent to remain neutral on whether dynamical shifts don't preserve solutions of NG_{NST}.
We thank an anonymous referee for pressing us on this important subtlety.

$$E^{(NG_{NST})} = -\frac{1}{8\pi} h^{ab} \nabla_a \varphi \nabla_b \varphi.$$

It's easily verified to be invariant exactly under Galilei-shifts: static and kinematic shifts don't alter the gravitational energy density. By contrast, due to the scaling in the gravitational potentials, dynamical shifts do: two NG_{NST} models related via dynamical shifts *differ on* their gravitational energy density.

Should this disconcert us? Clearly – no: The two NST models, related via kinematic (Γ) or dynamic shifts (Δ) describe *distinct* worlds. Consequently, gravitational energy density, set within NST, isn't a gauge-quantity – contra Dewar and Weatherall. That gravitational energy doesn't vary between such worlds is irrelevant.

In conclusion: NST *has* sufficient structure to ward off the spectre of gauge-dependence for gravitational energy density.

To be sure, NST is an objectionable space-time setting for NG. Via its unobservable absolute standard of rest, its dynamical and space-time symmetries don't align. This flouts Earman's adequacy conditions. Yet, one mustn't conflate the flaws of a *space-time setting* with the (alleged) shortcomings of a quantity – gravitational energy – *defined within this space-time setting*.

How does the situation look in space-time settings that amend this defect of NST? We next discuss Galilean space-time.

V.2.3 Galilean Space-time (GST)

GST ameliorates (some of) NST's shortcomings: it drops the assumption of absolute rest, i.e. the vector field σ^a. Thereby, one can pare down redundant structure. The points of space in GST's no longer persist: their diachronic identity is jettisoned. In NG_{GST}, one identifies all DPMs of NG_{NST} that differ only through Galilean shifts Γ.[107] Thus, GST retains an absolute standard

[107] In the case of static shifts, as discussed above, one may invoke sophisticated substantivalism, i.e. the denial that worlds are distinct that differ only with regard to which spacetime points exhibit which metrical properties (cf., for instance, Pooley, 2013, §7).
In the case of kinematic shifts, the symmetry arguably only motivates the search for a more perspicuous ontology that can metaphysically elucidate the identity of kinematically shifted worlds (Møller-Nielsen, 2017). This is

of straightness of paths between two events: whether a path is straight – a geodesic with respect to the flat derivative operator ∇_b – is an absolute matter of fact. (In modal language: Those DPMs of NG_{NST} in which the spatio-temporal paths of all possible test matter are parallel are identified as describing the same world.[108]) Contrariwise, as in NST, dynamically shifted DPMs remain distinct: in NG_{GST}, dynamical shifts *aren't* gauge-transformations.

What does this imply for gravitational energy density? The expression for gravitational energy density for NG_{GST} carries over from §2.3:

$$E^{(NG_{GST})} = -\frac{1}{8\pi} h^{ab} \nabla_a \varphi \nabla_b \varphi.$$

It didn't depend on σ^a, anyway. Under dynamical shifts it changes.

But as before, this is harmless: dynamically shifted DPMs describe distinct worlds. Hence, that they differ on their gravitational energy, doesn't render the latter gauge-variant. (To be sure, dynamically shifted DPMs are empirically indistinguishable. All relational quantities remain unaltered. So, an observer in one of several dynamically shifted worlds couldn't ascertain which is hers. This predicament may be metaphysically lamentable – but it's not a shortcoming of gravitational energy.)

As in the NG_{NST} case, Dewar and Weatherall's diagnosis of the gauge-dependence of gravitational energy is therefore unfounded in NG_{GST}.

The rebuttals of Dewar and Weatherall's claim so far may appear trivial. After all, GST - and a fortiori NST- arguably aren't the most perspicuous space-time settings for NG.[109] The analysis becomes more interesting for the two possible improvements on NG_{GST}, Maxwell-Huygens Gravity (NG_{MHST}) and Newton-Cartan Theory (NCT), respectively. We'll conclude this section with the former, before turning to the latter in §3.

V.2.4 Maxwell-Huygens spacetime (MHST)

provided by GST's transition from a 3-dimensional to the 4-dimensional picture of reality (cf., for instance, Maudin, 2012, pp. 54).

[108] This follows from the fact that the totality of geodesics on a manifold uniquely determine a derivative operator.

[109] It's all the more surprising that GST is the only space-time setting (apart from NCT) for NG which Dewar and Weatherall explicitly consider.

In light of his Corollary VI, Newton[110] may be credited with recognising the empirical indistinguishability of models of NG related via uniform accelerations, $\vec{x} \mapsto \vec{x}' = \vec{x} + \vec{d}(t)$ (Saunders, 2013). Here, $\vec{d}$ is a twice-differentiable function, representing an accelerational boost. Nonetheless, Newton persevered in his belief in absolute space. In rational (albeit historically incorrect, see Huggett & Hoefer, 2015, §6; Rynasiewicz, 2011) reconstructions, he is frequently (e.g. Maudlin, 2012, Ch. 2) imputed an invocation of an inference to the best explanation for inertial effects. Consider, for instance, the surface of a water-filled pail. It's (observably!) concave, if and only if the bucket is rotating. Is this rotation best conceptualised as rotation in absolute space, with the latter understood at least at the level of NG_{GST}? At first blush, it might appear so. But in fact, NG_{MHST} further whittles down NG_{GST}'s structure by exploiting the symmetry of the Poisson Equation under uniform accelerations. NG_{GST} only preserves an absolute sense of non-linear acceleration (equivalently[111]: rotation), evinced in inertial effects, such as in the above bucket experiment.

In NG_{MHST}, as we understand it (see below), one identifies NG_{GST} models related via uniform accelerations:[112] One stipulates that they describe the same world. In contrast to uniform accelerations in NST and GST, in MHST they *are* gauge-transformations.

Recall that within the geometric framework of classical space-times, derivative operators encode inertial structure. Hence, they define standards of accelerations. For MHST, one must thus identify those derivative operators that correspond to the same standards of non-rotational acceleration, up to uniform-accelerational transformations. This translates into the following condition for any two such standards of acceleration ∇ and ∇', and all unit, time-like vector-fields ξ^a (see Malament, 2012, pp. 263 for details):

$$\nabla^{[a}\xi^{b]} = 0 \Leftrightarrow \nabla'^{[a}\xi^{b]} = 0.$$

One can envision this condition as the requirement that the verdict whether trajectories of free particles in those space-times are not twisted be independent of the choice of the

[110] "If bodies, anyhow moved among themselves, are urged in the direction of parallel lines by equal accelerative forces; they will all continue to move among themselves, after the same manner as if they had been urged by no such forces" (Newton, 1729).

[111] To be sure: A time-dependent linear acceleration –a linear acceleration that is still an arbitrary function of time, i.e. of the form $\xi^b \nabla_b \xi^a = \alpha^a(t) + \beta(t)\xi^a$ – isn't equivalent to a rotation. But any non-linear acceleration is.

[112] Again, considering the Poisson equation only. Particle equations of motion will be considered below.

standard of acceleration. Such space-times are rotationally equivalent, with rotation understood as the twisting of possible free-fall trajectories.

Formally, one can now quotient out those of NG_{GST}'s KPMs which differ only by uniform accelerations (for details, see Weatherall, 2015). A KPM in NG_{MHST} thus takes the form of the following 6-tuple:

$$\langle \mathcal{M}, t_a, h^{ab}, [\nabla], \varphi, \varrho \rangle.$$

The only novel object, unfamiliar from GST is $[\nabla]$, the "standard of rotation". It's the equivalence class of flat, metrically compatible, rotationally equivalent derivative operators in GST:[113]

$$[\nabla] = \{\nabla' : \mathrm{R}'^{a}_{bcd} = 0 \,\& \nabla'_a t_b \& \nabla'_c h^{ab} = 0 \,\& [|\xi^a t_a| = 1 \Rightarrow (\nabla'^{[a}\xi^{b]} = 0 \;\Leftrightarrow \nabla^{[a}\xi^{b]} = 0)]\}.$$

Here, R'^{a}_{bcd} is the Riemann tensor associated with ∇'.[114]

It's straightforward to show that the Newton-Poisson Equation in GST, $h^{ab}\nabla_a\nabla_b\varphi = 4\pi\rho$, remains invariant under changes of rotationally equivalent derivative operators (Dewar, 2018). DPMs in NG_{MHST} can thus be obtained by identifying rotationally equivalent, but otherwise identical, DPMs of $NG_{GST.}$

To invest NG_{MHST} with empirical content, we still need equations of motion for matter under the influence of gravity. They group together the equations of motion for NG_{GST} within a standard of rotation, such that their (time-like, unit) solutions $\xi, \xi' \in T\mathcal{M}$ define accelerations (with respect to the *same* derivative operators) that differ only by linear accelerations: [115]

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[113] Speaking of an equivalence class of derivative operators, rather than (e.g.) defining a primitive standard of rotation (as does e.g. Weatherall 2017), invokes Dewar's 'sophistication' about symmetries (2018). A recent sceptical attitude towards said 'sophistication' can be found in Martens & Read (ms). Since we share the latters' scepticism, ultimately we would find it preferable to work with Weatherall's standard of rotation, rather than an equivalence class of operators. Nevertheless, for continuity with the literature, we set such concerns aside in the remainder of this article.

[114] Two rotationally equivalent derivative operators ∇ and ∇' in this class are related via $\nabla' = (\nabla, \eta^a t_b t_c)$ for some spacelike vector field η^a, satisfying $\nabla^b \eta^a = 0$ (Dewar, 2018, p.256).

[115] We wish to underscore that, given our cautious approach to theory identity, our version of NG_{MHST} *differs* from the ones, primarily considered in the literature (e.g. Dewar, 2018; Weatherall, 2017). Our version's ontology includes a gravitational field $\nabla^a\varphi$. By contrast, in Weatherall's (2017) treatment, "we (do not) need to interpret the gravitational potential or corresponding gravitational field, $\nabla\varphi$, as representing facts about force or a field-like entity" (p. 88). Similarly, in Dewar's treatment there is also no (privileged) choice of gravitational potential or gravitational field.

$$\{\xi \in T\mathcal{M}: \exists \nabla' \in [\nabla] \text{ such that } \xi^b \nabla'_b \xi^a + \nabla'^a \varphi = 0\}$$
$$= \bigcup_{\nabla' \in [\nabla]} \{\xi \in T\mathcal{M}: \xi^b \nabla'_b \xi^a + \nabla'^a \varphi = 0\}.$$

The (class of) equations of motion picking out this solution set, $\{\xi^b \nabla'_b \xi^a + \nabla^a \varphi = 0 : \nabla' \in [\nabla]\}$, is trivially invariant under uniform accelerations.

What might be candidates for gravitational energy in NG$_{\text{MHST}}$, $E^{(NG_{\text{MHST}})}$? The most natural one is defined as the equivalence class of all gravitational energy densities of rotationally equivalent NG$_{\text{GST}}$ models. As the action of two derivative operators upon a scalar is the same, $\nabla\varphi = \nabla'\varphi$, this equivalence class is well-defined: the gravitational energy densities of two Galilean spacetimes with rotationally equivalent derivative operators ∇' and ∇ coincide,

$$E' = -\frac{1}{8\pi} h^{ab} \nabla'_a \varphi \nabla'_b \varphi = -\frac{1}{8\pi} h^{ab} \nabla_a \varphi \nabla_b \varphi = E.$$

Consequently, within a model of NG$_{\text{MHST}}$, gravitational energy density is a well-defined quantity. It's *not* gauge-dependent.[116] (Note that while uniform accelerations are gauge transformations in NG$_{\text{MHST}}$, dynamical shifts, which also include a transformation of the potential, aren't.)

Again, Dewar and Weatherall's proclamation of the gauge-dependence of gravitational energy doesn't apply to NG$_{\text{MHST}}$. Like in NG$_{\text{GST}}$, that dynamically shifted models of NG$_{\text{MHST}}$ differ in their gravitational energy densities is benign: they describe distinct worlds.

One may, however, repudiate this formulation of NG within MHST for two reasons: its implausible conceptual prerequisites and its radicalness, respectively.

Firstly, it's unsatisfactory that in order to define MHST via an equivalence class of derivative operators of GST, one draws on structure that ultimately one *doesn't* attribute to the space-time (cf. Weatherall, 2017; Dewar, 2017; Martens & Read, ms).[117]

Compare the transition from NST to GST: there, the standard of absolute rest, represented by the time-like vector field σ^a, could simply be excised: it played only an otiose role in the

[116] Note that on Dewar's version of NG$_{\text{MHST}}$ the gravitational energy density *would* count as gauge-variant, as the potential would change.
[117] One could also rephrase this objection in terms of physical degrees of freedom. Concepts natural to a theory reflect these, as it were, carving nature at its joints. For NCT, the true physical quantities are $U(1)$-invariants – rather than boosts, parameterized by $U(1)$ (Teh, 2017).

formulation of NG.) Indeed, elsewhere Weatherall (forthc.) proffers an alternative characterisation of MHST without reference to derivative operators. Absent a derivative operator, though, how to define gravitational energy density? Evidently, the standard definition is no longer available in that case.

Gravitational energy density could well turn out not to be definable at all (as Dewar and Weatherall themselves admit)! That would certainly be grist to Dewar and Weatherall's mills – but for reasons other than those they cite. (It would be desirable to investigate whether gravitational energy density could be defined without derivative operators. We'll not pursue this, here, though.)

Elsewhere, Weatherall (forth., sect. 5) draws attention also to a second blemish of MHST: it's more revisionary than at first blush it appears. Forces as they figure on the l.h.s. of Newton's 2nd Law are absolute: they are formulated in terms of *one* derivative operator. It's unclear whether all of (non-gravitational) physics can be reformulated on MHST. Think, for instance, of the Abraham-Lorentz-Dirac force, describing the recoil force of accelerated charged particles due to radiation (see Rohrlich, 2007): it's manifestly *not* invariant under uniform accelerations. Hence, adopting MHST as the space-time setting for Newtonian physics necessitates a revision of the mathematical and conceptual foundations of much of classical physics. This may seem gratuitously radical.[118]

Another response to NG_{GST}'s redundancy is therefore appealing. In conjunction with the equivalence of inertial and gravitational mass, its symmetry under dynamical shifts motivates a geometrisation of NG: like in General Relativity, gravitational effects are absorbed into the space-time's non-flat inertial structure. The result is known as Newton-Cartan Theory (NCT). To this we turn next.

V.3. Newton-Cartan Theory

[118] The revisionary nature of MHST also crops up with respect to its interpretation. Recall that (considering the gravitational field equations only) two GST models correspond to the same Maxwell-Huygens spacetime, if and only if they differ merely up to uniform accelerations. Consequently, two DPMs of GST that, albeit rotationally equivalent, differ merely in their potentials, count as distinct. That raises the question of how to interpret the scalar in MHST: What is its ontological status? Is it a real physical field, on a par with, say, the electromagnetic one? On which space does it live?

We'll now investigate Dewar and Weatherall's claim that gravitational energy isn't well-defined in NCT. This section first (§3.1) reviews the basics of NCT. Next (§3.2), we expound why Dewar and Weatherall's arguments are specious. In §3.3, we try to fill the gap in their reasoning.

3.1 Geometrised NG

In this section, we review the basics of NCT, as contained in Trautman's Geometrisation Lemma and its converse Recovery Theorem (for all details, see Malament, 2012, Ch. 4.2).

In NCT, the gravitational potential of NG is absorbed into NCT's (non-flat) derivative operator. This is encapsulated in Trautman's Geometrisation Lemma.

Let $\langle \mathcal{M}, t_a, h^{ab}, \nabla_a \rangle$ be a Galilean (henceforth: "classical") spacetime. (The derivative operator ∇_a is assumed to be flat; its associated Riemann tensor vanishes, $\mathrm{R}^a_{bcd} = 0$.) Let furthermore φ and ϱ be smooth, real-value scalar fields on $\mathcal{M}$ which obey the Poisson Equation, $h^{ab}\nabla_a\nabla_b\varphi = 4\pi\varrho$. Finally, let $\widetilde{\nabla}_a = (\nabla, -\mathrm{t}_a\mathrm{t}_b h^{cd}\nabla_d\varphi)$.[119] Then, the following three propositions hold:

1. $\langle \mathcal{M}, t_a, h^{ab}, \widetilde{\nabla}_a \rangle$ is a classical spacetime.
2. $\widetilde{\nabla}_a$ is the (unique) derivative operator such that for all time-like curves on $\mathcal{M}$ with 4-velocity ξ^a: $\xi^a\widetilde{\nabla}_a\xi^b = 0 \Leftrightarrow \xi^a\nabla_a\xi^b = -h^{bc}\nabla_c\varphi$.
3. The Riemann curvature $\widetilde{\mathrm{R}}^a_{bcd}$ associated with $\widetilde{\nabla}_a$ satisfies
    a. the "geometrised" Poisson Equation $\tilde{R}_{ab} \coloneqq \widetilde{\mathrm{R}}^c_{acb} = 4\pi\varrho t_a t_a$,
    b. and the curvature conditions $\widetilde{\mathrm{R}}^{ab}{}_{cd} = 0$ & $\widetilde{\mathrm{R}}^{a\ c}_{\ b\ d} = \widetilde{\mathrm{R}}^{c\ a}_{\ d\ b}$.

The second proposition states an equivalence between geodesic/un-accelerated/inertial motion with respect to one derivative operator, and particular accelerated/non-inertial motion with respect to another: exactly those curves are geodesics with respect to $\widetilde{\nabla}_a$ that describe accelerated motion that is the result of the Newtonian gravitational force, with respect to ∇_a. In this sense gravity is geometrised - or rather "inertialised" (cf. Nerlich, 2013,

[119] That is (see Malament, 2012, Ch. 1.7): Let ∇ and ∇' be two derivative operators on a manifold $\mathcal{M}$. Then (following op.cit., p. 53), we'll write $\nabla' = (\nabla, C^a_{bc})$, iff they are related via a symmetric tensor field C^a_{bc}: For any tensor $\alpha^{a_1 \dots a_r}_{b_1 \dots b_s}$ of rank (r, s) on $\mathcal{M}$, $(\nabla'_m - \nabla_m)\alpha^{a_1 \dots a_r}_{b_1 \dots b_s} = \alpha^{a_1 \dots a_r}_{n b_2 \dots b_s} C^n_{m b_1} + \alpha^{a_1 \dots a_r}_{b_1 n b_3 \dots b_s} C^n_{m b_3} + \dots - \alpha^{d a_2 \dots a_r}_{b_1 \dots b_s} C^{a_1}_{md} - \alpha^{a_1 d a_3 \dots a_r}_{b_1 \dots b_s} C^{a_2}_{md} - \cdots$.

Ch. 9; Lehmkuhl, 2014, esp. §4): the deviation from inertial trajectories, defined via ∇_a, due to the gravitational force is reconceptualised as a manifestation of (non-flat) inertial structure, defined via $\widetilde{\nabla}_a$. (The interpretation of the curvature conditions shan't concern us here. Instead, we refer to Malament, 2012, Ch. 4.3.)

Via the Recovery Theorem, we can re-translate geometrised NCT gravity back into non-geometrised NG_{GST}.

Let the classical spacetime $\langle \mathcal{M}, t_a, h^{ab}, \widetilde{\nabla}_a \rangle$ satisfy the geometrised Poisson Equation $\tilde{R}_{ab} = 4\pi\varrho t_a t_a$ for some smooth scalar field ϱ on $\mathcal{M}$, and the Trautmann curvature conditions $\widetilde{\mathrm{R}}^{ab}{}_{cd} = 0$ & $\widetilde{\mathrm{R}}^{a}{}_{b}{}^{c}{}_{d} = \widetilde{\mathrm{R}}^{c}{}_{d}{}^{a}{}_{b}$. Then, in the neighbourhood of any point a real-valued scalar φ and a derivative operator ∇ exist, such that the following propositions hold:

1. ∇ is compatible with t_a and h^{ab}.
2. ∇ is flat. (Its associated Riemann tensor vanishes, $\mathrm{R}^{a}_{bcd} = 0$.)
3. For all time-like curves on $\mathcal{M}$ with 4-velocity ξ^a: $\xi^a \widetilde{\nabla}_a \xi^b = 0 \Leftrightarrow \xi^a \nabla_a \xi^b = -h^{bc}\nabla_c \varphi$.
4. φ satisfies the Poisson Equation: $h^{ab}\nabla_a\nabla_b\varphi = 4\pi\varrho$.

Via the Recovery Theorem, we can "de-geometrise" NCT spacetimes: geodesic/inertial motion with respect to $\widetilde{\nabla}$, which was force-free, is now re-conceptualised as accelerated/non-inertial motion with respect to ∇, subject to the gravitational force.

The de-geometrisation isn't unique. A second pair φ' and ∇' for which

$$h^{ab}\nabla_a\nabla_b(\varphi - \varphi') = 0 \;\&\; \nabla' = \left(\nabla, \mathrm{t}_a \mathrm{t}_b h^{cd}\nabla_d(\varphi - \varphi')\right)$$

also satisfies the conditions 1.-4. of the Recovery Theorem.

The transformations between any pair (φ, ∇) and (φ', ∇') that each satisfies the two non-uniqueness conditions are the dynamical shifts, mentioned in §2.2. Consequently, two models of NG_{GST} related via dynamical shifts are "de-geometrisations" of the *same* NCT spacetime. It has therefore been argued -e.g. by Pooley (2012, §6.1.1) or Knox (2014)- that the gravitational scalar and the derivative operator of ungeometrised NG –i.e. NG_{GST}- are merely gauge-dependent quantities; geometrised NG -i.e. NCT- provides a gauge-free formulation of NG. With its dynamical symmetries matching its spacetime symmetries, and hence conforming to Earman's adequacy conditions, NCT is a satisfactory theory of gravity.

In summary: NCT allows us to re-conceptualise gravitational effects as manifestations of non-flat spacetime geometry (inertial structure). Models of NG_{GST} related via dynamical shifts can be identified as the same NCT spacetime.

V.3.2 Dewar and Weatherall on gravitational energy in NCT

Let's now assess Dewar and Weatherall's principal argument against gravitational energy in NCT. Its logical form can be reconstructed as follows:

(1) The natural expression for gravitational energy in NG_{GST} isn't invariant under dynamical shifts.

(2) In NCT, one identifies those DPMs of NG_{GST} that are related via dynamical shifts as physically equivalent; they are gauge.

(3) *Therefore*, gravitational energy in NCT isn't gauge-invariant.

Our authors correctly observe (1) and (2). However, their conclusion -(3)- is objectionable for a simple reason: nowhere do Dewar and Weatherall *explicitly* define the object that is supposed to most naturally represent gravitational energy in NCT.

This is a crucial shortcoming. It renders their argument both formally and substantively incomplete. After all, Trautman's Geometrisation Lemma and Recovery Theorem (§3.1) only equip us with a translation between the 6-tuple $\langle \mathcal{M}, t_a, h^{ab}, \nabla_a, \varrho, \varphi \rangle$ of non-geometrised NG_{GST} quantities, and the 5-tuple $\langle \mathcal{M}, t_a, h^{ab}, \widetilde{\nabla}_a, \varrho \rangle$ of geometrised NCT quantities; both are silent on any other quantities.

For Dewar and Weatherall's above syllogism to *formally* go through, premise (1) needs to be superseded by

> (1') The (most natural) *NCT counterpart* of the Galilean gravitational energy isn't invariant under dynamical shifts.

With this, the conjunction of all three premises entails the conclusion:

$$(1')\&(2) \rightarrow (3).$$

But why believe that (1') is true? It's far from clear –as Dewar and Weatherall concede themselves- whether the NCT counterpart of Galilean gravitational energy *even exists* – and if it does, whether it indeed fails to be invariant under dynamical shifts. (To be sure, if either could be negated, this would be grist to Dewar and Weatherall's mills. Their *conclusion* would remain intact. But it would follow from *different reasons*: namely those against the existence of the most natural NCT counterpart of Galilean gravitational energy, rather than the gauge-dependence of an actually existing NCT gravitational energy.) In short: It's one thing to doubt the definability of gravitational energy; it's another to doubt its physical meaningfulness (or *well*-definedness). Dewar and Weatherall focus on the latter.

Even if one charitably grants that the meaning of "most natural candidate" is clear, one may impugn the very existence of an NCT counterpart of Galilean gravitational energy. As the Geometrisation Lemma discloses, Galilean gravitational energy contains terms absent in NCT. In the latter's DPMs, a gravitational potential doesn't appear; it has been absorbed by NCT's non-flat connection. Furthermore, Galilean gravitational energy is defined via the (flat) derivative operator of GST. Which derivative operator should then enter the NCT counterpart of Galilean gravitational energy? An intuitive choice would, of course, be NCT's (non-flat) derivative operator. But this is scarcely compelling.

If thus gravitational energy in NG_{GST} essentially hinges on terms absent in NCT, then why assume that it can be defined at all in NCT?

In conclusion: Unless the possible candidate for NCT's gravitational energy is explicitly defined, Dewar and Weatherall's criticism of the latter's (alleged) gauge-dependence forfeits much of its force.

To fill this lacuna, we'll now discuss various concrete options.

V.3.3. Candidates for gravitational energy in NCT

In the preceding section, we argued that Dewar and Weatherall's criticism of gravitational energy in NCT is vitiated by their lack of an explicit definition of gravitational energy in NCT. Here, we'll examine a number of natural candidates: 1. pseudotensors, 2. Komar energy, 3. Lorentz and Levi-Civita's proposal, 4. The Bel-Robinson tensor. 5. Pittsification. Rather than suffering from gauge-dependence, these proposals will be argued to be either not well-defined, or to yield trivial gravitational energy.

Dewar and Weatherall (2018, fn. 30) enjoin such an examination of explicit proposals. It has two kinds of merits. After all, in empirically equivalent theories, radically different objects can play the same role. (Think of Starobinski's original model of cosmic inflation (see, e.g., De Felice & Tsujikawa, 2010, sect. 2,3 for details.) In one formulation, the latter is driven by a scalar, hence arguably a matter field *on* spacetime. In an equivalent formulation, inflation is merely a manifestation *of* spacetime curvature deviating from what it should be according to GR.) Furthermore, comprehending the various possibilities in which a conceptually rich theory such as NCT can fail to exhibit a certain feature considerably enhances our understanding of it. In particular, this broadening of our repertoire of instruments is likely to pay off in comparing NCT to other theories in its theoretical vicinity, such as GR. (In the apt terms of Pitts (2017): Spacetime philosophy should aspire to "modal cosmopolitanism" - rather than "modal provincialism".)

V.3.3.1. Pseudotensors

In this subsection, we evaluate the natural NCT counterparts of the general-relativistic pseudotensors as possible candidates for gravitational energy. They are found to trivialise the latter.

The standard approach to gravitational energy in GR proceeds via the Noether theorems.[120] The absence of a (tractable, natural) Lagrangian or Hamiltonian formulation of NCT encumbers this road, though.[121]

One might, however, take the definitions of pseudotensors, as familiar from GR, and just *stipulate* their formal NCT analogues. What encourages such a procedure is that pseudotensors – at least in GR – arguably satisfy natural desiderata for local gravitational energy, e.g. a conservation law, the dependence only on first derivatives of the field variables, or the reduction to the familiar Newtonian potential energy in the weak-field limit (Dürr, 2018, §3.2).

[120] Historically too, this was Einstein's route – avant la lettre (Brading, 2005).

[121] In private correspondence, Nic Teh has conjectured that the non-existence of a Lagrangian or Hamiltonian formulation without Lagrange multipliers of NCT is even provable (cf. Hansen, Hartong & Obers, 2019).
It's straightforward to find a Lagrangian *with* suitable multipliers. But the latter are, of course, under-determined. Canonical gravitational energy-momentum for (i.e. the Noether current attributed to) the (non-flat) NCT metric would depend on the Lagrange multipliers, and hence would be ill-defined.

Following Goldberg (1958), an infinitely large class of pseudotensor densities (of arbitrary weights $n+1, n \in \mathbb{N}^{\geq 0}$) can be constructed as follows. (We restrict ourselves to mixed indices – one up, one down.)

$$\vartheta_\mu^{(n)\nu} = |g|^{\frac{n}{2}} \left\{ \vartheta_\mu{}^\nu + \frac{n}{2} U_\mu^{[\nu\sigma]} \partial_\sigma \ln|\mathrm{g}| \right\}$$

Here, $|g|$ denotes the modulo of the determinant of GR's metric. $U_\mu^{[\nu\sigma]}$ denotes a so-called super-potential. (The details needn't detain us here.)

For $n = 0$, we obtain the weight-one density of the Einstein-pseudotensor $t_\mu^{\ \nu}$:[122]

$$\vartheta_\mu^{(0)\nu} = \sqrt{|g|} t_\mu^\nu := 2\sqrt{|g|} G_\mu^\nu + \partial_\sigma \left(|g|^{-\frac{1}{2}} g_{\mu\lambda} \partial_\rho (|g| g^{\lambda[\nu} g^{\sigma]\varrho}) \right).$$

Together with the matter energy-momentum tensor $|g|^{\frac{n+1}{2}} T_\nu^\mu$, (of weight $n+1$), the pseudotensors –representing gravitational energy-momentum – form the system's *total* energy-momentum $\mathcal{T}_\mu^{(n)\nu} := |g|^{\frac{n+1}{2}} T_\nu^\mu + \vartheta_\mu^{(n)\nu}$. The latter satisfies the continuity equation:

$$\partial_\nu \mathcal{T}_\mu^{(n)\nu} = 0.$$

Albeit not a tensor equation, this continuity equation holds in all coordinate systems. Hence, total energy-momentum can be said to be (locally/differentially) conserved.

For the NCT counterparts to the general-relativistic pseudotensors, it's tempting to replace the general-relativistic metric in the above expressions by NCT's spatial or temporal pseudo-metric, h^{ab} and $t_{ab} = t_a t_b$ respectively. In fact, it can be shown (Andringa et al., 2011) that NCT doesn't admit of a non-degenerate metric with which the NCT connection is compatible. Hence, the subsequent discussion is without loss of generality.

However, due to their degeneracy, i.e. vanishing determinant, this is a non-starter: one can easily verify that the resulting NCT pseudotensors either are trivial or nor defined at all. The latter is the case for $\vartheta_\mu^{(0)\nu}$, i.e. $n = 0$;[123] the former is the case for $\vartheta_\mu^{(n)\nu}$s for $n > 0$.[124]

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[122] Alternate formulations can be found in e.g. Dirac (1975, Ch.31,32) or Ohanian & Ruffini (2013, A5).
[123] Dirac's affine form of the weight-1 Einstein pseudotensor density also yields a vanishing result for a singular metric.
[124] A slightly more interesting approach with the same negative result is the following. By evaluating the connection's action on a vector field, one can easily show that

$$\tilde{\nabla} = (\nabla, C^a_{bc}) \Leftrightarrow \tilde{\Gamma}^a_{bc} = \Gamma^a_{bc} + C^a_{bc}.$$

In conclusion: The natural NCT counterparts to GR's standard pseudotensor weights either are either ill-defined, or they yield a trivial notion of gravitational energy. While consonant with Dewar and Weatherall's conclusions, this result has nothing to do with a lack of gauge-invariance.[125]

V.3.3.2. Komar mass

This section is devoted to a plausible definition of total energy of NCT spacetimes via the Komar integral. Like pseudotensors, it trivialises gravitational energy.

The most natural path to a global notion of gravitational energy in GR proceeds via the Noetherian route or, equivalently, the Hamiltonian formalism. As mentioned in the preceding section, for NCT this path is blocked. For static spacetimes in GR, an alternative exists: the Komar integral. (In GR, it coincides with the Hamiltonian definition, see e.g. Poisson, 2004, Ch. 4.3.)

Consider a static spacetime, i.e. one with a(n asymptotically normalised) time-like Killing field ξ, satisfying $\nabla_{(a}\xi_{b)} = 0$. For such a spacetime, there exists a natural definition of "holding an object in place" via ξ's orbit (see Wald, 1984, pp. 285 for details). This gives rise to a likewise natural notion of acceleration with respect to this orbit. Via this acceleration, a force can be defined that an observer at infinity must exert in order to keep a unit mass in place. Analogously to the characterisation of the total energy of the electrostatic field in terms of its asymptotic properties, we thus arrive –after various manipulations, for which we refer to the

For the Newton-Cartan connection components, we therefore get

$$\tilde{\Gamma}^a_{bc} = \Gamma^a_{bc} - h^{an} t_b t_v \nabla_n \phi,$$

where Γ^a_{bc} denote the components of the flat connection (∇) of Galilei spacetime.

Now replace GR's connection components by those of the Newton-Cartan connection, i.e. $\tilde{\Gamma}^a_{bc}$ in Dirac's (1975, Ch.31) form of the Einstein pseudotensor. It's straightforward to verify that, due to the orthogonality conditions of the spatial and temporal metric, the result is ill-suited for representing gravitational energy: the resulting Newton-Cartan pseudotensor doesn't depend on the gravitational potential of the de-geometrised theory; it only depends on Γ^a_{bc}. In other words: the resulting Newton-Cartan pseudotensor *isn't* related to NCT's gravitational degrees of freedom. The same applies to the affine version of the Landau-Lifshitz pseudotensor (Landau & Lifshitz,1971, Ch. 96), when replacing in it the general-relativistic connection components by those of the above NCT connection.

[125] GR's pseudotensors are usually regarded as tainted by the problem of coordinate dependence (cf., for instance, Weyl, 1923, p. 273). By contrast, NCT's pseudotensors are free from that evil: whenever they are defined, the NCT pseudtensor densities vanish coordinate-independently. In the same vein, they are –albeit *trivially*- invariant under dynamical symmetries.

literature (ibid.) – at the following expression for the energy enclosed in the topological 2-sphere $\mathcal{S}_t$ in the hypersurface orthogonal to ξ:

$$E = -8\pi \lim_{\mathcal{S}_t \to \infty} \oint_{\mathcal{S}_t} d\sigma^{ab} \, \nabla_a \xi_b.$$

Here, $d\sigma^{ab}$ denotes the surface element on $\mathcal{S}_t$. This integral can serve as a definition of total energy in general-relativistic static spacetimes. It turns out to be conserved.

Given that NCT spacetimes are static in a natural sense,[126] it's now tempting to stipulate the NCT counterpart of the Komar integral as a candidate for the total energy of NCT spacetimes as well. To that end, one plausibly replaces the Killing field in the Komar expression's integrand by NCT's time covector, $\xi_a \to t_a$. This already suffices to trivialise the proposal: due to the compatibility condition of NCT's time pseudo-metric, $\nabla_a t_b = 0$, the NCT counterpart of the Komar integral vanishes. Consequently, the total energy of a NCT spacetime would be zero. Gravitational energy – understood as the energy left after subtracting the energy contributions of ordinary matter – would then always exactly counterbalance matter energy. This is implausible for reasons that we'll explain in the next subsection, in which we'll discuss Lorentz and Levi-Civita's proposal.

V.3.3.2. Lorentz and Levi-Civita's proposal

Lorentz and Levi-Civita proposed the Einstein tensor, $\mathrm{G}_{ab} = \mathrm{R}_{ab} - \frac{1}{2} R \mathrm{g}_{ab}$ (or, for reasons of dimensionality, $-\frac{1}{2\kappa}\mathrm{G}_{ab}$, with $\kappa := \frac{4\pi G}{c^4}$) as a representation of gravitational energy in GR (for details, see Cattani & DeMaria, 1993, sect. 5-11). Is this convincing for the NCT case? For reasons again both general and specific to NCT, we argue that this isn't the case.

Three facts commend Lorentz and Levi-Civita's proposal. (1) In contrast to pseudotensorial approaches, the Einstein tensor is a bona fide tensor. (2) It obeys a bona fide covariant conservation law: the contracted Bianchi identity, $\nabla_b G^{ab} \equiv 0$. The attendant total energy-momentum ${}_{(LLC)}\mathfrak{T}^{ab} := -\frac{1}{2\kappa} G^{ab} + \mathrm{T}^{ab}$, satisfies both an ordinary and covariant continuity equation, $\partial_b\left({}_{(LLC)}\mathfrak{T}^{ab}\right) = \nabla_b\left({}_{(LLC)}\mathfrak{T}^{ab}\right) = 0$. (3) The Einstein tensor is the exact *gravitational* counterpart of the *matter* energy-momentum tensor: whereas the latter is defined

[126] That is: Its defining partial differential equations are elliptic. Hence, information about variations in a region propagates instantaneously.

variationally as $T_{ab} = -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}\mathcal{L}_{(m)}\right)$, one obtains the Einstein tensor (up to a proportionality factor) by replacing the matter Lagrangian by the purely gravitational Einstein-Hilbert Lagrangian,

$$G_{ab} \propto \frac{1}{\sqrt{|g|}}\frac{\delta}{\delta g^{ab}}\left(\sqrt{|g|}R\right).$$

The first two features carry over to NCT. The third one, however, doesn't: the absence of a natural Lagrangian formulation of NCT's *full* gravitational sector (cf. Hansen, Hartong & Obers, 2019) − including the two Trautmann conditions imposed on curvature − weakens the analogy between the Einstein tensor and the matter energy-momentum tensor.

But there are stronger reasons to question Lorentz and Levi-Civita's proposal: physical implausibility and vacuity, respectively (cf. Pauli, 1981, fn 180-181). Firstly, consider the Einstein Equations in vacuum. This, on Lorentz and Levi-Civita's proposal, yields *vanishing* gravitational energy, $G_{ab} = 0$. But that's counterintuitive: since the Einstein tensor is constructed from traces of the Riemann tensor, a solution of the vacuum Einstein Equations has in general non-vanishing Weyl structure.[127] The latter encapsulates gravitational radiation. Prima facie, one would expect it to possess gravitational energy – contrary to Lorentz and Levi-Civita's proposal (cf. Dürr, 2018 for a critique). Equally implausibly, it purports that there are no differences between gravitational energy in the exterior of a static and, say, rotating black hole, respectively: in either case, gravitational energy would be zero. For NCT, the objection needs to be slightly adapted. NCT's Poisson Equation is elliptic. Hence its solutions can't propagate. In that sense, there is of course no gravitational radiation. Still, one would expect different NCT spacetimes with non-vanishing Weyl structure − i.e. different homogenous solutions of the Poisson Equation − to differ in their gravitational energy. (Recall that the Weyl tensor measures tidal deformations in the shape of extended spacetime regions.)

Besides such doubts regarding its physical plausibility, it seems mysterious and contrived that, on Lorentz and Levi-Civita's proposal, any matter energy-momentum is exactly counterbalanced by gravitational energy (in the GR case): in *all* possible spacetimes, the *total* energy always vanishes, $-\frac{1}{2\kappa}G_{ab} + 2\kappa T_{ab} = 0$. It's elusive what positing such an entity would

[127] Dewar and Weatherall (2018, sect. 4) show that for NCT spacetimes, one can indeed define a (non-trivial) Weyl tensor (cf. Ehlers & Buchert, 2009; Wallace, 2016; Duval, Gibbons & Horvathy, 2017).

help *explain*. As Levi-Civita conceded in a letter to Einstein, the proposal is sterile in that "[…] the energy principle would lose all its heuristic value, because no physical process (or almost none) could be excluded a priori. In fact, [in order to get any physical process] one only has to associate with it a suitable change of the [gravitational field]". For NCT, this sterility is exacerbated by the fact that the Einstein tensor reduces to the Ricci tensor, and that the latter vanishes for mixed indices,

$$^{(NCT)}G_a^b \equiv {}^{(NCT)}R_a^b \equiv 0.$$

In other words: Lorentz and Levi-Civita's proposal yields only a trivial gravitational energy-momentum flux along some direction ξ^a: $^{(NCT)}G_a^b \xi^a \equiv 0$.

In conclusion: The Einstein tensor isn't suited for representing gravitational energy in both GR and NCT; it lacks physical informativeness and plausibility. The issue of gauge-dependence under dynamical shifts doesn't arise in any form.

Let's turn next to another tensorial proposal, Bel and Robinson's superenergy tensor.

V.3.3.3 The Bel-Robinson Tensor

In this subsection, we examine the NCT counterpart of the Bel-Robinson tensor as a candidate for NCT's gravitational energy.

Recall the energy-momentum tensor of electrodynamics:

$$4\pi T^{\mu\nu}_{(em)} = F^{\mu}_{\ \lambda} F^{\lambda\nu} - \frac{1}{4} g^{\mu\nu} * \left(F^{\kappa\lambda}\right) * (F_{\kappa\lambda}),$$

with the Faraday tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, and its dual $*\,(F^{\mu\nu}) = \epsilon^{\mu\nu\kappa\lambda} F_{\kappa\lambda}$. (We use the latter – rather than the non-dual – in the second term of the energy-momentum tensor to render its structural similarity with the Bel-Robinson Tensor more transparent.) In an analogous manner, one can construct a tensor from the Riemann tensor,[128] mimicking the symmetric electromagnetic energy-momentum tensor (see Garecki, 2018 for details). The result is the so-called "superenergy tensor":

[128] This may be motivated by Synge's suggestion that GR's gravitational field is represented by the Riemann curvature tensor (cf. Lehmkuhl, 2008a for a critical discussion). That is: According to Synge, one should view the Riemann tensor as the GR counterpart to the Faraday/field strength tensor – a view backed up by the perspective from the fibre bundle formalism (Weatherall, 2016c).

$$T^{abcd} := R^{aefc}R^{b}{}_{ef}{}^{d} +* (R^{aefc}) * \left(R^{b}{}_{ef}{}^{d}\right)$$
$$= R^{aefc}R^{b}{}_{ef}{}^{d} + R^{aefd}R^{b}{}_{ef}{}^{c} - \frac{1}{2}g^{ab}R^{efgc}R_{efg}{}^{d}.$$

(Here, $*$ denotes the usual dual operation: $*(R_{abcd}) = \frac{1}{2}\epsilon_{abef}R^{ef}_{\ \ cd}$.) Bel and Robinson proposed it as a candidate for gravitational energy in GR.

As a consequence of the Bianchi identities (and hence, independently of the Einstein Equations), its covariant divergence vanishes:

$$\nabla_a T^{abcd} \equiv 0.$$

Note that due to the Einstein Equations, in vacuum the Riemann tensor can be replaced by the Weyl tensor. The latter encodes gravitational degrees of freedom that can propagate through vacuum. In light of this, the Bel-Robinson tensor seems apt for describing energy associated with gravitational radiation.

What makes it of particular interest is that the Bel-Robinson tensor appears in the expansion of the Einstein pseudotensor at a point, when evaluated in normal coordinates for some other point (see So, Nester & Chen 2009 for details).

Due to the flatness of NCT spacetimes (in the sense of $R^{ab}{}_{cd} = 0$, §3.1), a non-trivial Bel-Robinson tensor in NCT must be defined as a tensor of rank (1,3):

$$^{(NCT)}T^{i}{}_{klm} := R^{i}{}_{abl}R^{b}{}_{mk}{}^{a} +* \left(R^{i}{}_{abl}\right) * \left(R^{b}{}_{mk}{}^{a}\right),$$

with the Riemann tensors (and their duals), associated with the NCT connection. As the Bianchi identities also hold in NCT, also $\nabla_i {}^{(NCT)}T^{i}{}_{klm} \equiv 0$ obtains.

However, $^{(NCT)}T^{i}{}_{klm}$ isn't a convincing proposal for gravitational energy in NCT for reasons both general and specific to NCT.

Generally (and like in GR), it has the dimensions $length^{-4}$. So, neither the Bel-Robinson tensor nor any of its powers have the right dimension, unless one introduces a novel constant of nature. But this seems ad-hoc.

Moreover, the Bel-Robinson tensor is linked to *differences* in pseudotensorial gravitational energy (and hence, on a standard interpretation of pseudotensors: to differences in gravitational energy simpliciter), rather than to the latter directly (ibid.). So, its physical interpretation would presuppose a non-trivial notion of pseudotensorial gravitational energy. But as we saw in §3.3.2, the most immediate NCT counterparts to pseudotensors are vacuous.

In conclusion: As a proposal for gravitational energy in NCT, the Bel-Robinson tensor is both formally, as well as in absence of its connection to non-vacuous pseudotensorial gravitational energy, unsuitable. Contra Dewar and Weatherall, gauge-variance isn't the issue here, though.

We conclude our perusal of candidates for gravitational energy in NCT with a non-tensorial proposal, due to Pitts.

V.3.3.3. Pittsification

Pitts (2010) has recently propounded an astute solution to the problem of coordinate/gauge-dependence of pseudotensors in GR: take your favorite pseudotensor, say the Einstein pseudotensor $\vartheta_a^{\ b}$, and declare the *totality* of its values in all possible coordinate systems (at neighbourhood of a point) *one* object. Symbolically:

$$\left\{(\forall \text{ coordinate systems CS})\left(\vartheta_\mu^{\ \nu}\right)_{CS}\right\}.$$

It has (uncountably) infinite components. Each corresponds to the pseudotensor's value in one possible coordinate system.

There are two ways to transfer this idea to NCT. The first one takes the NCT counterparts of pseudotensors, and "Pittsifies" them as in Pitts' original proposal for GR. But this is of little interest, as the NCT counterparts of pseudotensors are either trivial or not defined (§3.3.1).

More auspicious is another option. It starts from *NG_{GST}'s* gravitational energy. As described in §3.2, a DPM in NCT $\mathfrak{M}$ can be de-geometrised into an equivalence class of GST models $GST_\alpha(\mathfrak{M})$ for some index set $\alpha \in \mathcal{A}$. For any two $\alpha, \alpha' \in \mathcal{A}$, the models $GST_\alpha(\mathfrak{M})$ and $GST_{\alpha'}(\mathfrak{M})$ differ only up to dynamical shifts. Now Pittsify the gravitational energies of all these $GST_\alpha(\mathfrak{M})$s. This yields the (Pittsified) NCT gravitational energy, symbolically:

$$E(\mathfrak{M}) \coloneqq \{(\forall \alpha \in \mathcal{A}) E[GST_\alpha(\mathfrak{M})]\}.$$

Each component of this object corresponds to one possible GST de-geometrisation. By construction, it's gauge-invariant under dynamical shifts. (Recall: De-geometrisations of an NCT spacetime are all related via dynamical shifts.)

Pittsification welds together into one well-defined, formal object the gravitational energies of those GST spacetimes that correspond to the same NCT spacetime. It's not obvious, though, that it provides a satisfactory representation for gravitational energy in NCT: firstly, its conceptual prerequisites seem alien to NCT; secondly, one may have qualms about its physical meaningfulness.

The Pittsified NCT gravitational energy is constructed from the gravitational energies of those NG_{GST} spacetimes the geometrisation of which yields the same NCT spacetime. On the one hand, this yields a formally well-defined object – even a geometric one.[129] On the other hand, one may wonder: is it legitimate to introduce into a theory quantities built from terms that belong to, and are meaningful only within, a *different* theory? That is: Are we allowed to use quantities prima facie intelligible only in NG_{GST} in order to define a quantity supposedly meaningful in NCT?

Perhaps such a worry might be allayed by the thought that the *individual* de-geometrised NG_{GST} spacetimes lack meaning in NCT; only their *totality* accrues it. Consider the gauge-quantities of electromagnetism, the 4-potentials. By themselves, they don't possess physical significance, either (perhaps setting aside potential subtleties for the Aharonov-Bohm effect); only a suitable combination of them − i.e. the Faraday tensor − does. By analogy, one might argue that only the Pittsified NCT gravitational energy as a whole is meaningful; its individual components – the NG_{GST} quantities – aren't. One could counter by questioning the whole procedure: isn't Pittsification too cheap a trick to procure gauge-invariant quantities? Finding gauge-invariant quantities is a formidable task in ongoing research in (non-Abelian) gauge theories. One would like more than a *merely* formal object: how to ensure that the Pittsified gravitational energy *actually* possesses physical significance?[130] (Consider, by analogy, the Pittsification of the electromagnetic 4-potentials, i.e. the infinite-component object made up of all 4-potentials in all possible gauges. In a formal sense, it's evidently gauge-independent. One would baulk, however, at attributing it physical content, as expressed in the electromagnetic fields.)

[129] If one is willing to extend the standard meaning of geometrical objects to objects with infinitely many components (Pitts, 2010, §1-2).

[130] This is squarely related to the question of inferences from symmetries to reality: declaring the physical equivalence between symmetry-related models of a theory remains merely formal and verbal, unless a metaphysically perspicuous explication of the corresponding ontological picture is forthcoming (Møller-Nielsen, 2017).

In conclusion: Via Pittsification, we can define a formal candidate for gravitational energy of a NCT spacetime from the gravitational energies of its corresponding NG_{GST} de-geometrisations. Reasons to object to this proposal *don't* include gauge-dependence; rather, they consist in doubts about its physical significance and conceptual adequacy.[131]

V.4. Discussion

Dewar and Weatherall (2018, pp. 26) conclude their paper with "an important lesson for how to understand energy in geometrized theories. […] (T)here is a deep relationship between the classical notions of energy, work, force, and inertia. Energy is a measure of the ability to do work […] But in theories in which gravitation is 'geometrized' in the sense that gravitation is understood as an inertial effect in curved spacetime, we should not think of gravitation as a force at all – and so, in particular, it is not the sort of thing that does work. To the contrary, work makes sense only as a measure of the deviation from inertial motion over some distance."

Our discussion illustrates this insight in slightly more detail.[132] It shows explicitly that *non-geometrised* variants of NG in the above sense –NG_{NST} and NG_{GST} –*do* allow for a well-defined notion of gravitational energy. Contrariwise, for NG in spacetime settings where inertial structure has absorbed gravity –MHST and NCT – gravitational energy faces several obstacles.[133] The status of a prima facie central concept such as the energy associated with Newtonian gravitational degrees of freedom crucially depends on how Newtonian Gravity is *interpreted*.

Dewar and Weatherall make a farther-reaching suggestion: "[…] we should understand the energy density of Yang-Mills fields [including electromagnetism, the authors] as *relative* to some background structure – namely, the inertial structure determined by the spacetime metric in general relativity" (p. 27, their emphasis). We concur with this. But it's worthwhile

[131] This criticism mirrors the one mounted against Pittsification of pseudotensors in GR (Dürr, 2018, §3.3).

[132] We plan to complement our and Dewar and Weatherall's results by an investigation from the view-point of teleparallisation (for a conceptual introduction, see Knox, 2011). Recently, Teh & Read (2018) have shown that the Trautman Recovery Theorem is an instance of teleparallelisation. It will be interesting to study whether further illuminating insights into gravitational energy in NCT and NG can be gained by applying the machinery of teleparallelisation.

[133] It's important that the notion of geometrisation relevant here is specific – the absorption of gravitational effects into inertial structure. *Other* notions of geometrisation (see Lehmkuhl, 2008b, Ch. 9) don't seem relevant. Reichenbach's geometrised toy unification of gravity and electromagnetism (Giovanelli, 2016), for instance, admits of the standard GR electromagnetic energy-stress tensor.

stressing that it implies a minor rectification of Dewar and Weatherall's conclusion, cited above: it's less the geometrisation of gravity itself –the absorption of gravitational effects by inertial structure- that is responsible for the difficulties in defining gravitational energy. Rather, it's the existence of (sufficiently rich) inertial structure *simpliciter* that seems to be a prerequisite for a meaningful (or at least, robust, cf. Read, 2017) definition of field energies.[134] In fact, one may construe the main problem diagnosed in §2.4 for gravitational energy for NG_{MHST} as a violation of this requirement: MHST's inertial structure is too impoverished to even allow us to define gravitational energy; for that, the derivative operator had to be imported from GST.

Our discussion also emphasised an additional difficulty for gravitational energy for NCT: the absence of a natural Lagrangian (or Hamiltonian) formulation. Despite the similarities with respect to geometrizing gravity, this makes its status *more* precarious than in GR. Energy is arguably a cluster concept. But it wouldn't be too much of a stretch, either, to regard the definition of energy within the Lagrangian/Hamiltonian framework as the primary meaning of energy in field theories.[135] Hence, we propose, not only will the comparison with Yang-Mills theories be rewarding, as Dewar and Weatherall suggest; it will likewise be illuminating to investigate the status of field energies in non-Lagrangian theories.

[134] This point seems pertinent for all theories in which the notion of inertial trajectories becomes questionable. In fact, this is the case for Bohmian Mechanics (see Acuña, 2016): energy is no longer a fundamental concept.

[135] Note that Brown (2020, fn. 15; pp. 9) points out a *naïve* view about the intrinsic connection of energy (conservation) and "homogeneity of time" (i.e. invariance under time translations) arguably isn't viable. Prima facie, a cluster concept view affords more flexibility.

This chapter:

In those versions of Newtonian Gravity in which gravity isn't fully geometrised, defining local gravitational energy seems unproblematic. Once we move to the fully geometrised Newton-Cartan theory, however, we encounter problems akin to those in General Relativity: gravitational energy seems "geometrised away" then, too.

The next chapter:

Does this finding extend to other, non-Newtonian theories of gravity? We'll next inspect a precursor theory of General Relativity – Nordström's theory of gravity of 1913.

VI. Nordström Gravity

Abstract:

This chapter re-examines Nordström's scalar theory of gravity (NG) – arguably the most convincing relativistic theory of gravity before the advent of General Relativity. It exists in two different forms. In Nordström's original one (1913), NG appears to describe a scalar gravitational field on Minkowski spacetime. In Einstein and Fokker's (1914) version, NG seems to be a spacetime theory: it reconceptualises gravitational effects as manifestations of non-Minkowskian inertial structure. Both variants of NG give rise to three contradictory verdicts on the status and validity of fundamental principles: the Weak Equivalence Principle, the existence of gravitational energy, and energy conservation. Given the putative equivalence of both variants of NG, this ambiguity seems paradoxical to the spacetime realist. I'll proffer a resolution from the perspective of integrable Weyl geometry: the paradoxes rest on the failure to recognise a more apposite spacetime setting for NG. With this new spacetime setting in place, both variants of NG, which prima facie look more like distinct theories, can be identified as notational variants of each other.

Key words: General Relativity, Scalar Theories of Gravity, Weyl Geometry, Gravitational Energy, Energy Conservation, Theory Equivalence, Spacetime Realism

VI.1. Introduction

In 1907, Einstein was commissioned with a review article on the burgeoning special theory of relativity (SR) and its applications. Only one branch of physics proved recalcitrant – gravity (broached in section V). Here, the quest for a relativistic theory of gravity commences. Six years later, in 1913, Nordström (1913b) proffered the first satisfactory candidate. Already a few months later, in his lecture on the status quo of relativistic theories of gravity, Einstein (1913) acknowledged that Nordström's theory indeed met his four desiderata for such theories:

1. Energy and momentum should be conserved.
2. In closed systems, inertial and gravitational mass should be equal.
3. The theory should respect covariance under Lorentz transformations.
4. In a homogenous gravitational field, the laws of nature should take a form independent of the absolute value of the gravitational field.

In Nordström's original[136] presentation of his theory, NG_N, gravity is prima facie represented by a scalar on Minkowski spacetime. It obeys the simplest relativistic generalisation of the Poisson Equation of Newtonian Gravity.

Alas, NG_N is as dead as a dodo: it predicts no bending of light, and a perihelion *lag*. This verdict of NG_N's empirical inadequacy comes with hindsight, however (cf. Giulini, 2008). It was hoped that observations during eclipses in 1914 (cf. Einstein, 1913, §10 and subsequent discussion) would adjudicate between Nordström's and competitor theories, including Einstein's own *Entwurf* theory (Einstein & Grossmann, 1913) predicting light-bending. The outbreak of the Great War dashed such hopes. (Five years later, in 1919, Eddington eventually confirmed the effect. By then, of course, General Relativity (GR), was enjoying pride of place in gravitational physics.) Around 1913, Mercury's anomalous precession played only a subordinate role for theoretical developments (cf. Norton, 2005, §16). The reasons are twofold. On the one hand also the *Entwurf* theory –the only other theory considered by Einstein which also satisfied his above desiderata – failed to account for it. Only GR achieved a convincing explanation (see e.g. Earman & Janssen, 1993; Renn & Schemmel, 2012). On the other hand, owing to

[136] Nordström (1912, 1913a) had in fact devised a cognate of his theory a year earlier. It soon proved objectionable in several regards (see Norton, 2005, sect. 6-8). I'll restrict my discussion throughout the paper to his second theory.

uncertainties regarding our solar system, Mercury's perihelion advance was largely deemed peripheral.

Why bother about NG_N, then except for historical reasons? Notwithstanding its empirical shortcomings, NG_N merits the philosopher's attention. It's of interest for studying gravitational waves, the Equivalence Principle and the meaning of units.

First, NG_N is the first coherent theory that admits of gravitational radiation (cf. Laue, 1917; Shapiro & Teukolsky, 1993).[137] Vis-à-vis GR (e.g. Misner, Thorne & Wheeler, 1973, Ch. 18), NG_N is qualitatively different, though: it displays gravitational monopole (rather than: quadrupole) radiation, associated with massless spin-0 (rather than: spin-2) particles.[138] NG_N can therefore serve as a toy model for conceptual questions revolving around gravitational waves – without GR's mathematical intricacies (cf. Watt & Misner, 1999 for a similar argument for another toy scalar theory).

Secondly, NG_N implements (some versions of) the Equivalence Principle – a feature of NG_N Einstein praised, as reported above. A whole section will be dedicated to the issue below. Ironically (and unrealised by Einstein), though, his *Entwurf* theory defies the principle's original (1907, p. 454) version: uniform acceleration and a homogenous gravitational field *don't* produce the same physical effects − not even in infinitesimal approximation (Norton, 2018, §13). NG_N therefore naturally lends itself as a test case for different formulations of the Equivalence Principle (see e.g. Norton, 1985; Dieks, 2006; Di Casola, 2014; Lehmkuhl, 2019).

Thirdly, under the continued pressure of Einstein (Norton, 1993; 2005, §6-8), Nordström was forced to posit "running units" in his theory. In a natural sense (to be unpacked below), units in NG_N are no longer fixed: the dimensions and durations of physical systems and processes depend on the gravitational potential. This raises questions pertinent to the status of units and dimensions in general (cf. Bunge, 1971). In particular, one may ponder: how to identify two theories, if one of them is predicated on "running" and the other on "fixed" units (cf.

[137] Trailblazers for the idea of gravitational waves were Heaviside, Poincaré and Abraham (cf. Kennefick, 2007, Ch. 2). To be sure: The first (historically contingent) *actual* prediction of gravitational waves based on a coherent theory − GR − came in 1916 by Einstein (op.cit, esp. Ch. 3-5 for a historical survey).

[138] Note also in this regard the comparison with Scalar Tensor Theories. Like NG_N, they also exhibit scalar radiation modes (see e.g. Faraoni & Capozziello, 2011, Ch. 5.4). But, by contrast to NG_N, the corresponding (spin-2) graviton is *massive*, and of *dipole* nature in leading order.

Dicke, 1962; Quiros et al., 2013)? This feature of NG_N will lie at the heart of the subsequent analysis.

In 1914, Einstein and Fokker showed that, besides Nordström's scalar field variant, NG also possesses an equivalent, purely metric representation – NG_{EF}. The nature of this equivalence is noteworthy in its own right, as we'll see. But also by itself, NG_{EF} is conceptually remarkable (little appreciated at the time, though): it's a generally covariant *spacetime* theory of gravity, conforming to the Action-Reaction Principle!

First and foremost, like GR, NG_{EF} admits of a geometric interpretation (in the sense of strength-3 geometrisation in Lehmkuhl, 2009): gravitational effects can be reconceptualised as a manifestation of NG_{EF}'s non-flat spacetime geometry itself.[139] Deepening our understanding of NG_{EF}'s geometrisation of gravity and its ramifications will thus also shed light on more complex geometrised theories of gravity, such as GR or f(R) Gravity.

Secondly, NG_{EF} is fully generally covariant[140]; its formulation doesn't presuppose a preferred set of coordinate systems (pace Einstein and Fokker (1914, §2), whose remarks suggest otherwise). Ironically, at this point, Einstein had already (temporarily) abandoned general covariance (Stachel, 1989; Norton, 1993x, 2005; Janssen & Renn, 2015). (Grossmann and Einstein's *Entwurf* field equations are of limited covariance, see Norton, 1984; Renn, 2007.) This is, of course, the story of Einstein's misadventures with the Hole Argument (e.g. Norton, 1984, 1987; Stachel, 2014).

Thirdly, Einstein used to tout satisfaction of the Action-Reaction Principle as the principal epistemological improvement of GR over SR (see Brown & Lehmkuhl, 2013): GR's chronogeometric and inertial structure affect matter, while at the same time, matter affects −backreacts upon − them in turn. *Despite* the presence of an absolute element in the sense of Anderson (1967, 1971; cf. Friedman, 1983, Ch. 2,3) − viz. the Minkowskian light-cone structure − NG_{EF} satisfies the Action-Reaction Principle: its full metric (more precisely: the latter's conformal factor) is determined by matter. NG_{EF} therefore is a promising test case for different

[139] As Lehmkuhl (2014) has shown, Einstein himself rejected this interpretation, Instead, he championed a *unificatory* interpretation of GR: the gravito-inertial field, represented by the connection, subsumes both gravity and inertia.
My subsequent discussion will refer only to GR's nowadays more orthodox spacetime interpretation. Prima facie, NG_{EF} seems no less amenable to Einstein's unificatory interpretation than GR.
[140] This means that Nordström Gravity (in the Einstein-Fokker formulation) is indeed the first generally covariant spacetime theory (pace Norton, 2019, sect.2) – albeit perhaps unrecognised.

definitions of absolute objects (see e.g. Pitts, 2006), general covariance (see e.g. Norton, 1993, Pooley, 2009), or background independence (see e.g. Read, 2016; Teitel, 2019).

NG's preceding features are intimately intertwined with three conundrums ("Mysteries"). Upon them the present chapter will focus. The Mysteries concern the following three facts:

- **(M1)**: Prima facie, NG_N violates the Geodesic Principle: test particles don't follow NG_N's spacetime geodesics; a universal force deflects them from straight (Minkowskian) trajectories. In NG_{EF}, by contrast, test particles follow NG_{EF}'s (non-Minkowskian) geodesics. The Geodesic Principle seems satisfied.

- **(M2)**: In NG_N, one can assign the scalar degree of freedom an energy-stress tensor. Thus, NG_N appears to admit of a meaningful notion of gravitational energy.
  In NG_{EF}, by contrast, in defining gravitational energy one encounters similar challenge as in GR: qua the absorption of gravitational degrees of freedom by NG_{EF}'s spacetime structure, gravitational energy becomes a compromised notion both formally and interpretatively.

- **(M3)**: In NG_N, only the sum total of energy of (non-gravitating) matter and gravity is conserved. In NG_{EF}, by contrast, only the energy of (non-gravitational) matter is.

(M1), (M2) and (M3) implicate core concepts of modern gravitational theories. This renders their prima facie ambiguous, theory-dependent status unsettling. Both the Geodesic Principle –as a generalisation of Galilei's Law of Inertia- and total energy conservation arguably are fundamental principles. As such, whether they hold or not should be an absolute fact, independent of one's choice of either NG_N or NG_{EF}. Likewise, energy is a pivotal notion in all of physics. Moreover, whether a physical quantity bears a certain amount of energy (and a fortiori: whether it bears energy at all) should be an absolute fact, quite apart from one's predilection for either NG_N or NG_{EF}. The point is aggravated by the fact that both are standardly referred to as *reformulations* of each other: this suggests that they are representational variants of the *same* theory. But then how can the status of the Geodesic Principle, gravitational energy and energy conservation hinge on the *conventional* choice of a theory's representation?

In what follows, I'll debunk the Three Mysteries (M1)-(M3) as merely apparent. They originate in a facile characterisation of NG's spacetime structure, in conjunction with a spurious identification of a scalar gravitational field as NG_N's referent.

My analysis will proceed as follows. In **§2**, I'll review the basics and interpretation of NG in both Nordström's original version (**§2.1**), as well as Einstein and Fokker's field theoretic version (**§2.2**). **§3** casts into sharper relief the above Three Mysteries: (M1), pertaining to energy conservation (**§3.1**), (M2), pertaining to gravitational energy (**§3.2**), and (M3), pertaining to total energy conservation (**§3.3**). In **§4**, I'll clarify the empirical and theoretical relationship between NGN and NGEF. Thereby, the paradoxes constituted by (M1)-(M3) will be resolved. **§5** summarises some wider-reaching lessons from the analysis, given in this chapter.

VI.2. Nordström Gravity

In this section, I'll first (§2.1) outline Nordström's original formulation of NG, NG_N. I'll then turn to Einstein and Fokker's geometric reformulation, N_{EF} (§2.2).[141]

VI.2.1. Nordström's Theory

Nordström's 1913 formulation, NG_N, casts NG in the guise of a theory of a scalar gravitational field on Minkowski spacetime.

That is, NG_N's kinematically possible models (KPMs) consist of the quintuple

$$\langle \mathcal{M}, \boldsymbol{\eta}, \boldsymbol{\nabla}, \phi, \boldsymbol{\Psi} \rangle. \tag{1}$$

Here, $\mathcal{M}$ and $\boldsymbol{\eta}$ denote the manifold of spacetime points (events), and the Minkowski metric on it, respectively.[142] The derivative operator ("connection") $\boldsymbol{\nabla}$ is assumed to be compatible with it: $\boldsymbol{\nabla} \cdot \boldsymbol{\eta} \equiv 0$. The scalar $\phi: \mathcal{M} \to \mathbb{R}$ encodes the gravitational degree of freedom. The generic $\boldsymbol{\Psi}$ represents NG_N's (non-gravitational) matter fields.

[141] NG's historical development, as well as their role in the genesis of GR are recounted in Pais (1982), Ch. 13; Isaakson (1985) and Norton (1993, 2005).
[142] I'll adopt the sign convention such that in Cartesian coordinates, the Minkowski metric simplifies to $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$.

NG_N's dynamically possible models[143] (DPMs) are picked out from its KPMs via a field equation for ϕ and the non-gravitational matter dynamics for $\mathbf{\Psi}$.[144]

The former is furnished by a nonlinear inhomogeneous wave-equation on a Minkowski background. It relates the gravitational scalar and the matter degrees of freedom:[145]

$$\Box\phi = -\frac{4\pi}{1+\phi}T. \qquad (2)$$

Here, $\Box \coloneqq \boldsymbol{\nabla}\cdot\boldsymbol{\nabla} \equiv \eta_{ab}\nabla^a\nabla^b$ denotes the special-relativistic d'Alembertian wave-operator. On the r.h.s., $T := tr_{\boldsymbol{\eta}}\{\boldsymbol{T}\} \equiv \eta_{ab}T^{ab}$ denotes the trace of the energy-stress tensor of (non-gravitational) matter. This incorporates Einstein and Grossmann's earlier argument (1913, §1.7) that one should utilise T as the source density for a scalar theory of gravity.

Two remarks on $\boldsymbol{T}$ are in order. First, energy-stress tensors were introduced in the context of the relativistic theory of stressed continua by Laue only two years earlier, in 1911 (see Norton, 2005, §9 for details). Note, however, that the modern *variational* definition of energy-stress tensors was developed later by Hilbert (1915). Below, bridging their historical and modern usage, all energy-stress tensors will be defined variationally. Secondly, the energy-stress tensor on the r.h.s. of eq. (2) *isn't* the standard special-relativistic energy-stress tensor: it depends on the scalar, as we'll see shortly (thanks to the greater clarity of the variational definition of energy-stress).

Let's now turn to NG_N's matter sector. Historically, it was confined to test matter in the form of (uncharged) massive particles (and, by continuity considerations, perfect fluids, see Deruelle & Sasaki, 2011). At this juncture, a systematic extension to more general types of matter will have to be postponed to §4.3.2.

It will be convenient to formulate NG_N via an action principle. (I'll roughly follow Einstein, 1913, §3). NG_N's total action for $\nu = 1, \ldots, N$ particles with the respective proper times τ_ν, (affinely parameterised) worldlines $\boldsymbol{X}_{(\nu)}$ and masses $m_{(\nu)}$, is given by

[143] Van Fraassen (1980, p. 53) identifies a model of a theory as "any structure which satisfies the axioms of [that] theory". Following common practice in the spacetime philosophy literature, I'll henceforth identify models in Van Fraassen's sense with DPMs.

[144] For simplicity, I'll use Planck units throughout: $G = c = \hbar = 1$ (see e.g. Wald, 1984, Appendix F for a conversion table).

[145] For ease of comparison with Newtonian Gravity, the scalar is chosen such that in NG_N's Newtonian limit, $\phi \to 0$.

$$S_{tot}\left[\boldsymbol{\eta},\phi,\boldsymbol{X}_{(\nu)}\right]=-\frac{1}{8\pi}\int d^4x\sqrt{|\eta|}(\nabla\phi)^2-\sum_{\nu}m_{(\nu)}\int d\tau_\nu$$
$$-\textstyle\sum_\nu m_{(\nu)}\int\phi d\tau_\nu. \qquad (3)$$

It encompasses three components. The purely gravitational, free-field contribution is supplied by $S_g[\boldsymbol{\eta},\phi]:=-\frac{1}{8\pi}\int d^4x\sqrt{|\eta|}(\nabla\phi)^2$, where $(\nabla\phi)^2:=tr_{\boldsymbol{\eta}}\{\boldsymbol{\nabla}\phi\otimes\boldsymbol{\nabla}\phi\}$. The free-particle contribution is imported from SR: $\sum_\nu m_{(\nu)}\int d\tau_\nu$. The third term, $-\sum_\nu m_{(\nu)}\int\phi d\tau_\nu$, encapsulates the field-particle interaction term. Together, the free-particle contribution and the interaction term form the effective matter action, $S_m\left[\boldsymbol{\eta},\phi,\boldsymbol{X}_{(\nu)}\right]:=\int(1+\phi)\sum_\nu m_{(\nu)}d\tau_\nu$. Crucial for NG_N is the fact that the gravitational scalar $(1+\phi)$ couples non-minally (viz. directly) and universally to the gravity-free matter Lagrangian density $\sum_\nu m_{(\nu)}d\tau_\nu$ (more on this below).

Impose now Hamilton's Principle, $0=\delta S_{tot}\equiv\delta\phi\frac{\delta S_{tot}}{\delta\phi}+\delta\boldsymbol{X}_{(\nu)}\cdot\frac{\delta S_{tot}}{\delta\boldsymbol{X}_{(\nu)}}$, with the independent variables ϕ and $\boldsymbol{X}_{(\nu)}$. We then obtain the field equation for the scalar and the equations of motion for the particles. Both can be expressed economically via the energy-stress tensor (cf. Giulini, 2008; Uzan & Deruelle, 2014, Ch.10):

$$\boldsymbol{T}(x):=-\frac{2}{\sqrt{|\eta|}}\frac{\delta}{\delta\boldsymbol{\eta}}S_m=\sum_\nu\frac{m_{(\nu)}}{\sqrt{|\eta|}}\int(1+\phi)\delta^4(x-X_{(\nu)}(\tau_\nu))\,\dot{\boldsymbol{X}}_{(\nu)}\otimes\dot{\boldsymbol{X}}_{(\nu)}d\tau_\nu. \qquad (4)$$

This allows us to rewrite the matter action as:

$$S_m=-\int d^4x\sqrt{|\eta|}tr_{\boldsymbol{\eta}}\{\boldsymbol{T}\}=:-\int d^4x\sqrt{|\eta|}T. \qquad (5)$$

$\frac{\delta S_{tot}}{\delta\phi}=0$ then implies the gravitational field equation (2) for the scalar:

$$\Box\phi=-4\pi(1+\phi)^{-1}T. \qquad (6)$$

From $\frac{\delta S_{tot}}{\delta\boldsymbol{X}_{(\nu)}}=0$, the equations of motion for matter ensue:

$$\boldsymbol{\nabla}\cdot\boldsymbol{T}=T\boldsymbol{\nabla}\,ln(1+\phi). \qquad (7)$$

Eq. (5) and (6) are NG_N's constitutive equations (for test matter).

For static weak fields, they reduce in leading order to the Newtonian case. Eq. (2) becomes the Poisson Equation for Newtonian Gravity:

$$\Delta\phi = 4\pi\varrho, \tag{8}$$

with the mass density $\varrho(\vec{x}) = \sum_{\nu} m_{(\nu)}\, \delta^3(\vec{x} - \vec{x}_{(\nu)})$. Eq. (7) becomes the equation of motion for particles subject to the Newtonian gravitational force:

$$m_{(\nu)} \frac{d^2}{dt^2} \vec{x}_{(\nu)} = -m_{(\nu)} \frac{\partial}{\partial \vec{x}} \phi\Big|_{\vec{x}_{(\nu)}}. \tag{9}$$ [146]

Outside the Newtonian regime, NG_N's equations of motion yield the 4-forces acting on particles. For non-vanshing T, one obtains (following e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 8.8):

$$\frac{D}{D\tau_\nu} \boldsymbol{U}_{(\nu)} \coloneqq \boldsymbol{\nabla}_{\boldsymbol{U}_{(\nu)}} \boldsymbol{U}_{(\nu)} = -\boldsymbol{\nabla}\, ln(1+\phi) + \frac{d}{d\tau_\nu} ln(1+\phi)\, \boldsymbol{U}_{(\nu)} \tag{10}$$

Without the first term on the r.h.s., this describes a (non-affinely parameterised) geodesic of Minkowski spacetime (see e.g. ibid., Ch. 3.17). Through a suitable redefinition of the affine parameter, one can eliminate it. Hence, under the influence of gravity, massive particles are accelerated by $-\boldsymbol{\nabla} \ln(1+\phi)$. In the corresponding 4-force equation, gravity acts like a universal force: It distracts massive particles from geodesic trajectories. *Massless* particles, however, such as a photon fluid (recall that for a radiative fluid: $T \equiv 0$), one may expect not to be affected: in the limit $m_\nu \to 0$, the r.h.s of eq. (7) vanishes; massless particles follow Minkowskian geodesics. Note also that for a constant scalar, $\phi = const.$, eq. (10) reduces to the geodesic equation on Minkowski spacetime.

The universality of Nordströmian Gravity stems from the direct coupling of NG_N's scalar to the matter variables in eq. (5). It results in a ϕ-dependence of measurable, "natural distances" (Einstein, 1913) – as opposed to merely "coordinate distances": in the presence of a gravitational field, the length L_0 of a rod, or the time units T_0 elapsed during periodic processes (other than light-clocks) varies as $L = (1+\phi)L_0$ and $T = (1+\phi)^{-1}T_0$, respectively.

VI.2.2. Einstein-Fokker Theory

[146] Thus, NG_N provides an eliminative explanation of the equality of *Newtonian* inertial and gravitational mass (see e.g. Beckermann, 2008, Ch. 8): The latter is reduced to the inertial masses $m_{(\nu)}$ (cf. Weatherall, 2011).

Let's turn to Einstein and Fokker's presentation of the theory – NG_{EF}. It casts NG in a purely metric form. NG_{EF} is naturally (but not historically) interpreted as a theory of a (non-flat) spacetime.

Einstein and Fokker (1914) showed that NG_N, as introduced in §2.1, admits of a mathematically equivalent representation. Their approach is motivated by three steps. (For simplicity, I'll consider a 1-particle system.) First, express NG_N's matter action (for test-particles) via the new line-element $d\tilde{s} \coloneqq \sqrt{|(1+\phi)^2 \boldsymbol{\eta}(\boldsymbol{dx}, \boldsymbol{dx})|}$:

$$S_m = \int (1+\phi)\, ds = \int d\tilde{s}. \tag{11}$$

We can conceive of $d\tilde{s}$ as the line element of the *effective* metric $\boldsymbol{g} \coloneqq (1+\phi)^2 \boldsymbol{\eta}$. The latter is the metric to which the matter dynamics adverts: it measures NG_N's "natural distances", monitored by rods and clocks. $\boldsymbol{g}$ thus represents the NG's effective gravitational degrees of freedom.

Secondly, following Einstein and Grossmann, Einstein and Fokker opt for the trace of the energy-stress tensor, $T[\boldsymbol{g}, \boldsymbol{\Psi}] \equiv tr_{\boldsymbol{g}}\{\boldsymbol{T}[\boldsymbol{g}, \boldsymbol{\Psi}]\}$ as the source term for their gravitational field equation. (The notation $\boldsymbol{T}[\boldsymbol{g}, \boldsymbol{\Psi}]$ signifies the fact that the energy-stress tensor is defined relative to the metric $\boldsymbol{g}$, cf. Lehmkuhl, 2011**.** This is relevant for the variational definition of the energy stress tensor, $\boldsymbol{T}[\boldsymbol{g}, \boldsymbol{\Psi}] \coloneqq -\frac{2}{\sqrt{|g|}} \frac{\delta}{\delta \boldsymbol{g}} S_m$.)

In their third step, Einstein and Fokker prescribe that the field dynamics be only second order in first derivatives of $\boldsymbol{g}$, but linear in second derivatives. This fixes the possible dynamical term for $\boldsymbol{g}$: it must be a constant multiple of the Ricci scalar $R[\boldsymbol{g}]$, associated with $\boldsymbol{g}$ (Vermeil, 1917[147]).

Combining all three steps, one arrives at the following stipulation for NG_{EF}'s field equation:

$$R[\boldsymbol{g}] = \lambda T[\boldsymbol{g}, \boldsymbol{\Psi}], \tag{12}$$

for some coupling constant λ. (The latter is determined to be -24π via the Newtonian limit.)

More axiomatically, one postulates that NG_{EF}'s KPMs consist in the quadruple

$$\langle \mathcal{M}, \boldsymbol{g}, {}^{(g)}\boldsymbol{\nabla}, \boldsymbol{\Psi} \rangle. \tag{13}$$

[147] I'm unaware of an earlier publication of this result. It's unclear to me whether Einstein and Fokker anticipated this result, or whether they had access to an unpublished proof.

As before, $\mathcal{M}$ and $\boldsymbol{\Psi}$ denote the manifold of events, and the matter degrees of freedom, respectively. Now, however, $\boldsymbol{g}$ is a generic Lorentzian manifold, i.e. of signature $(-,+,+,+)$. ${}^{(g)}\boldsymbol{\nabla}$ is its associated Levi-Civita connection (compatible with it, ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{g} = \boldsymbol{0}$).

NG$_{EF}$'s DPMs are picked out by imposing two algebraic curvature conditions for the gravitational sector, and the equations of motion for the matter sector.

The first curvature condition is the global requirement that $\langle \mathcal{M}, \boldsymbol{g} \rangle$ be Weyl-flat:

$$\boldsymbol{C}[\boldsymbol{g}] = \boldsymbol{0}. \qquad (14)$$

Here, $\boldsymbol{C}[\boldsymbol{g}]$ denotes the Weyl tensor associated with $\boldsymbol{g}$, i.e. the trace-free part of the latter's Riemann tensor, $\boldsymbol{Riem}[\boldsymbol{g}]$ associated with ${}^{(g)}\boldsymbol{\nabla}$. That is, the components of the Weyl tensor are defined as (see e.g. Weinberg, 1972, pp. 145):

$$C_{abcd} = R_{abcd} + \frac{1}{3} g_{a[c} g_{b]d} R - g_{a[c} R_{d]b} + g_{d[c} R_{d]ba}. \qquad (15)$$

Condition (14) imposes a "prior geometry" (Misner, Thorne & Wheeler, 1973, §17.6): It compels the metric to be "conformally flat", i.e. of the form $\boldsymbol{g} \equiv \sqrt[4]{\frac{|g|}{|\eta|}}\boldsymbol{\eta}$.[148]

NG$_{EF}$'s second curvature condition supplies the dynamics for the metric residual degrees of freedom. They are essentially encoded in the Ricci curvature. Relating the latter's trace and that of the energy-stress tensor (as the source), we obtain NG$_{EF}$'s dynamical evolution for $\boldsymbol{g}$:

$$R[\boldsymbol{g}] = -24\pi T[\boldsymbol{g}, \boldsymbol{\Psi}]. \qquad (16)$$

For test matter, the energy-stress tensor $\boldsymbol{T}[\boldsymbol{g}, \boldsymbol{\Psi}]$ is procured via the "minimal substitution rule" (Wald, 1984, p. 70), familiar from GR, i.e. by making the following replacements in the special-relativistic expression:

[148] Geometrically (for all details see e.g. Malament, 2012, Ch. 1.9), this means that the spacetime's light-cone/causal structure is Minkowskian: vectors that are time-like (space-like) with respect to $\boldsymbol{g}$ are also time-like (space-like) with respect to $\boldsymbol{\eta}$, and vice versa. Furthermore, all angles between vectors defined with respect to $\boldsymbol{\eta}$ are same as those defined respect to $\boldsymbol{g}$. (A cloud of incoherent dust illustrates this: in a conformally flat spacetime manifold, its *shape* doesn't vary across it; only its *volume* can change (see e.g. Carroll, 2004, pp. 167; Poisson, 2004, Ch. 2 for details).
Recall, however, that flat and merely conformally flat spacetimes don't share the same geodesic structure: whilst they agree on which curves (up to re-parameterisation) count as *null*-geodesics, they differ on which curves count as *time-like* geodesics.

$$\begin{cases}\boldsymbol{\eta}\\ {}^{(\eta)}\boldsymbol{\nabla}\end{cases} \to \begin{cases}\boldsymbol{g}\\ {}^{(g)}\boldsymbol{\nabla}\end{cases}.^{149} \tag{17}$$

For instance, a perfect fluid as test matter with energy density ϱ and pressure P the energy-stress tensor reads

$$\boldsymbol{T}^{(pf)} = (\varrho + P)\boldsymbol{u}\otimes\boldsymbol{u} + P\boldsymbol{g}. \tag{18}$$

With this, we can formulate NG_{EF}'s equations of motion for matter. They are given by the vanishing covariant divergence of the energy-stress tensor

$$^{(g)}\boldsymbol{\nabla}\cdot\boldsymbol{T}^{(pf)} = 0. \tag{19}$$

From this, one educes that NG_{EF}'s inertial structure is given by the connection $^{(g)}\boldsymbol{\nabla}$: Freely falling test particles, including photons, trace out the geodesics associated with it. (The argument, e.g. Hobson, Efstathiou & Lasenby (2006), Ch. 8.8 carries over verbatim from GR.) Due to metric compatibility, such geodesic paths also extremise the line element $\int\sqrt{|\boldsymbol{g}(\boldsymbol{dx},\boldsymbol{dx})|}$.

N_{EF}'s field equations (14) and (16) for $\boldsymbol{g}$ now turn out to be mathematically equivalent to NG_N's field equation (2) for $\boldsymbol{g}$'s conformal factor, i.e. $1+\phi \coloneqq \sqrt[8]{|g|}$. That is: A DPM of NG_{EF}, $\langle\mathcal{M},\boldsymbol{g},{}^{(g)}\boldsymbol{\nabla},\boldsymbol{\Psi}\rangle$, determines a DPM of NG_N, $\langle\mathcal{M},\boldsymbol{\eta},{}^{(\eta)}\boldsymbol{\nabla},\sqrt[8]{|g|/|\eta|}-1,\boldsymbol{\Psi}\rangle$; conversely, a DPM of NG_N, $\langle\mathcal{M},\boldsymbol{\eta},{}^{(\eta)}\boldsymbol{\nabla},\phi,\boldsymbol{\Psi}\rangle$ determines a DPM of NG_{EF}, $\langle\mathcal{M},(1+\phi)^2\boldsymbol{\eta},{}^{((1+\phi)^2\eta)}\boldsymbol{\nabla},\boldsymbol{\Psi}\rangle$.

How to interpret NG_{EF}? Like GR, NG_{EF} is purely metric (cf. Will, 1993, esp. Ch. 3). There exists only one metric. It encapsulates all information about the gravitational degrees of freedom. All inner products that enter the matter dynamics are taken relative to $\boldsymbol{g}$ (rather than to $\boldsymbol{\eta}$, or any other metric). Consequently, $\boldsymbol{g}$ is invested with chronogeometric significance: ideal clocks survey the proper time $d\tilde{\tau} = \sqrt{|\boldsymbol{g}(\boldsymbol{dx},\boldsymbol{dx})|}$. In NG_{EF}, "naturally measured" and "coordinate" quantities in Einstein's sense coincide.

It therefore seems apt to interpret NG_{EF} in analogy to GR − as a geometric theory of gravity (cf. Friedman, 1983, esp. Ch. V; Stachel, 2002; Lehmkuhl, 2009, Part III; Weatherall, 2017b, §6): gravitational effects are expressed via the spacetime geometry, represented by the triple

[149] This precept is little more than a pragmatic rule of thumb. It's known to be ambiguous. Nor is its compatibility with (some versions of) the Strong Equivalence Principle clear (cf. Di Casola, Liberati & Sonego, 2015, sect. IV; Read, Lehmkuhl & Brown, 2017).

$\langle \mathcal{M}, \boldsymbol{g}, {}^{(g)}\boldsymbol{\nabla} \rangle$. That is: NG_{EF} reduces gravity to dynamical, non-Minkowskian chronogeometry and inertial structure, represented by $\boldsymbol{g}$ and ${}^{(g)}\boldsymbol{\nabla}$, respectively (Nerlich, 2013; Lehmkuhl, 2014, esp. sect. 2).[150]

In summary: NG_N is set up as a theory about a scalar field on Minkowski spacetime. It's governed by a non-linear Klein-Gordon equation. The gravitational potential's direct and universal coupling to matter elicits a dependence of measurable length and durations on the scalar. By contrast, in analogy to the Einstein field equations, NG_{EF} is set up as a theory about spacetime: Gravitational effects exhibit non-Minkowskian geometry chronogeometry and inertial structure. Each model of NG_N defines a model of NG_{EF}, and vice versa.

Let's now home in on a puzzling fact about NG: at first blush, it appears to depend on which variant one adopts −NG_N or NG_{EF}− whether certain putatively fundamental principles hold. This is in tension with either a naïve understanding of the physical equivalence between NG_N and NG_{EF}, or a naïve realism about spacetime.

VI.3. Crisis: Three Mysteries

In this section, I'll examine whether the following is true in our two variants of NG, reviewed in §2:

1. Does the Geodesic Principle hold? (§3.1)
2. Does gravitational energy exist? (§3.2)
3. Is total (gravitational *cum* matter) energy conserved? (§3.3)

NG_N and NG_{EF} appear to give different responses. This prima facie paradoxical equivocation constitutes NG's titular three mysteries.

VI.3.1 First Mystery: Geodesic Principle

The first paradox concerns spacetime geodesic motion: in NG_{EF}, it counts as inertial/force-free motion; in NG_N, it doesn't.

[150] I won't embroil myself in the controversy over whether spacetime is a fundamental entity, or whether it's reducible to more fundamental (matter) degrees of freedom. The above interpretation of NG_{EF} (and of GR) remains neutral in the debate between advocates of the "dynamical" and "geometric" approach to spacetime (e.g. Brown & Read, forth.).

Following Einstein (Lehmkuhl, 2014; cf. DiSalle, 2009; Petkov, 2012), one may regard the Geodesic Principle[151] (GP) as a generalisation of Galilei's Law of Inertia to curved spaces: A (test) body's spatiotemporal trajectory is a geodesic of the spacetime manifold. GR satisfies the GP (see e.g. Di Casola, Libertati & Sonego, 2015; Lehmkuhl, 2019).[152]

NG_N, by contrast, doesn't. If its spacetime is Minkowskian, according to the GP, test particles under the should follow Minkowskian geodesics – straight lines. The second term on the r.h.s. of eq. (10) prevents this: it deflects massive particles from Minkowksian geodesics.

Contrariwise, the GP's status in NG_{EF} is exactly like in GR: test particles *do* trace out NG_{EF}'s spacetime geodesics – those associated with ${}^{(g)}\boldsymbol{\nabla}$. The GP holds.

This finding presents is puzzling: how can the GP as a generalised Law of Inertia – and hence as a fundamental principle – hold in one formulation of NG (viz. NG_{EF}), but not hold in the other (viz. NG_N)? Isn't it an objective fact, independent of the *conventional* representation of a theory, whether a force is exerted on a particle or not?

This I'll dub the first mystery of NG – (M1).

VI.3.2. Second Mystery: Status of gravitational energy

NG's second paradox concerns the status of gravitational energy: NG_N's gravitational scalar has a standard energy-stress associated with it; in NG_{EF}, by contrast, scepticism about the meaningfulness of gravitational energy-stress is apposite.

Prima facie, NG_{EF}'s gravitational energy-stress appears unproblematic. The purely gravitational action for its scalar is a standard Klein-Gordon scalar (e.g. Wald, 1984, Appendix E1):

$$S_g[\boldsymbol{\eta}, \phi] = -\frac{1}{8\pi}\int d^4x\sqrt{|\eta|}\,(\boldsymbol{\nabla}\phi)^2. \tag{20}$$

[151] I forgo the label "Weak Equivalence Principle". Thereby, I hope to preempt possible confusions that allusion to the Equivalence Principle tends to engender.
Norton (2005, fn. 29; cf. Di Casola, Libertati & Sonego, 2015) writes: "The equality of inertial and gravitational mass, and the uniqueness [read: universality, P.D.] of free fall are distinct from the principle of equivalence." Norton also notes (p.22) that in the historical debate between Einstein and Nordström, both use *different* versions of the Equivalence Principle (cf. Norton, 1985).

[152] In a certain limit sense, the GP in GR can even be generalised to *extended* massive and massless bodies, subject to certain energy conditions (Di Casola, Liberati & Sonego, 2014; Geroch & Weatherall, 2018; Weatherall, 2018; cf. Tamir 2012, 2013).

From this, energy-stress for NG_N's scalar is defined canonically as[153]

$$\boldsymbol{\Theta} := -\frac{2}{\sqrt{|\eta|}}\frac{\delta}{\delta\boldsymbol{\eta}}S_g[\boldsymbol{\eta},\phi] = -\frac{1}{8\pi}\left(\boldsymbol{\nabla}\phi\otimes\boldsymbol{\nabla}\phi - \frac{1}{2}(\boldsymbol{\nabla}\phi)^2\boldsymbol{\eta}\right). \tag{21}$$

Noether's Theorem (alternatively: eq. (7)) guarantees that $\mathbf{T}$ and $\boldsymbol{\Theta}$ *together* are conserved: $\boldsymbol{\nabla}\cdot(\boldsymbol{T}+\boldsymbol{\Theta}) = \mathbf{0}$.

The extant literature, both historical and contemporary (e.g. Pais, 1982, p.234; Norton, 2005, pp. 57; Giulini, 2008, sect. 6.2; Gourgoulhon, 2012, p. 717) concurs with this stance: $\boldsymbol{\Theta}$ is identified as gravitational energy.

In NG_{EF}, things are more delicate (and not considered in the historical material). One difficulty is that NG_{EF}'s above (historical) presentation employs no Lagrangian. Hence, NG_{EF}'s counterpart to NG_N's canonical gravitational energy-stress $\boldsymbol{\Theta}$ isn't obvious.

Three remedies spring to mind: the canonical energy-stress obtained from a naïve Lagrangian formulation, the NG_{EF} counterparts of general-relativistic pseudotensors, and Einstein tensor for NG_{EF}'s metric. None, I submit, is persuasive.

By dint of a Lagrange multiplier λ_a^{bcd} with the symmetries of the Weyl tensor, a suitable Lagrangian density for NG_{EF} is easily constructed (Deruelle, 2011, sect. VIII):

$$\tilde{\mathfrak{L}}[g,\lambda] = \sqrt{|g|}\left(R[g] + \lambda_a^{bcd}C^a_{bcd}[g]\right). \tag{22}$$

Here, $\mathrm{R}[\mathrm{g}]$ and $C^a_{bcd}[\mathrm{g}]$ denote the Riemann scalar and the Weyl tensor of the (a priori unspecified) Lorentzian metric g, respectively.

Variation of $\int d^4x\,\tilde{\mathfrak{L}}$ with respect to both the metric and the Lagrange multiplier entails NG_{EF}'s vacuum field equations (14) and (16). One may now be tempted to define gravitational energy-stress variationally, analogously to matter energy-stress:

$$\tilde{\boldsymbol{\Theta}} := -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta\boldsymbol{g}}\int d^4x\,\tilde{\mathfrak{L}}. \tag{23}$$

The result, however, depends on the Lagrange multiplier λ_a^{bcd}. The latter is under-determined. This mars $\tilde{\boldsymbol{\Theta}}$'s physicality: $\tilde{\boldsymbol{\Theta}}$ isn't a well-defined physical quantity.

[153] Recall that for a scalar field, the canonical (Noetherian) energy-stress automatically coincides with the Hilbertian (variational) energy-stress.

A second approach to gravitational energy-stress in NG_{EF} draws inspiration from general-relativistic pseudo-tensors (see e.g. Goldberg, 1958; Trautman, 1962). In GR, they are the most widely considered candidates for gravitational energy-stress (see e.g. Misner, Thorne & Wheeler, 1973, §20).

For concreteness, consider the Einstein pseudotensor. In Dirac's (1996, Ch. 32) form it reads:

$$16\pi\sqrt{|g|}\vartheta_a{}^b[\boldsymbol{g}] \coloneqq \left(\Gamma^b_{cd} - \delta^b_c\Gamma^e_{ce}\right)\partial_a\left(\sqrt{|g|}g^{cd}\right) - \sqrt{|g|}\delta^b_a g^{cd}\left(\Gamma^e_{cd}\Gamma^f_{ef} - \Gamma^f_{ce}\Gamma^e_{df}\right). \quad (24)$$

Here, $\Gamma^a_{bc} = \begin{Bmatrix} a \\ bc \end{Bmatrix}_{\boldsymbol{g}}$ denote the connection coefficients of ${}^{(g)}\boldsymbol{\nabla}$, i.e. the Christoffel symbols of $\boldsymbol{g}$'s Levi-Civita connection.

The Einstein pseudotensor accords with intuitive desiderata for gravitational energy. For instance, it obeys the continuity equation $\partial_b\left(\sqrt{|g|}(\vartheta_a{}^b + T^b_a)\right) = 0$, commonly interpreted as expressing energy conservation. Like the energy-stress of other relativistic fields, it's built from only up to first derivatives in the field variable (here: the metric). Furthermore, it reduces to the gravitational energy of Newtonian theory.

Now define $\vartheta_a{}^b[\boldsymbol{g}]$ in NG_{EF} by substituting the general-relativistic metric by NG_{EF}'s metric $\boldsymbol{g}$. (Via the variational formulation given in §4.2, the canonical energy-stress associated with $\boldsymbol{g}$ turns out to *be* the general-relativistic pseudotensor, evaluated for NG_{EF}'s $\boldsymbol{g}$. In particular, the continuity equation needn't be postulated separately.) The result inherits the merits commending pseudotensors in GR as candidates for gravitational energy.

Unfortunately, pseudotensors thus imported from GR also inherit the familiar flaws. As in GR, pseudotensors in NG_{EF} defy a straightforward interpretation: they are "viciously coordinate-dependent" (Pitts, 2009, p. 16). That is: Only under affine transformations do equations featuring pseudotensors remain invariant. The symmetries of pseudotensors, and those of the spacetime don't align. In particular, they don't transform like 3-tensors under generic purely spatial transformations, tantamount to merely conventional re-labellings of points in space (Horský & Novotný, 1969, p. 431; cf. also Stachel, 2002, p. 4). In other words: Pseudotensors aren't geometric objects in the sense of Anderson (1967, 1971). In more detail, the covariance group for pseudotensors is the group of linear transformations $GL(1,3)$. NG_{EF}'s spacetime symmetry group, on the other hand, is the conformal group $C(1,3)$ (Pitts, 2018). The latter comprises besides two linear subgroups (corresponding to Poincaré transformations and dilatations/rescalings), also a *non-linear* subgroup, (corresponding to so-called special conformal transformations, see e.g. Schottenloher, 2008 for details). This mismatch between the symmetries of the pseudotensors and those of NG_{EF}'s spacetime renders the physical significance of pseudotensors questionable (cf. Duerr, 2018b §3.2; 2019). One may discard them as gauge-quantities.[154]

The same problem afflicts non-tensorial objects more generally. As candidate objects for representing gravitational energy, they are indeed ineluctable in purely metric theories. According to a recent theorem by Curiel (2018), there exists no tensor that satisfies the natural desiderata for gravitational energy-stress, apart from the Einstein tensor. (The proof doesn't hinge on the Einstein Equations. Curiel's result equally applies to NG_{EF} and metric theories in general.).

This leads to a third candidate for NGE_{EF}'s gravitational energy-stress: what about the Einstein tensor itself, $\mathbf{G} \coloneqq \mathbf{Ric} - \frac{1}{2}R\boldsymbol{g}$ for NG_{EF}'s metric $\boldsymbol{g}$?[155]

At first blush, the proposal augurs well. $\mathbf{G}$ is a bona fide tensor. Due to the Bianchi identity, it also satisfies a covariant conservation law: $\boldsymbol{\nabla} \cdot \mathbf{G} \equiv 0$. Furthermore, in contrast to GR (e.g. Wald, 1984, Ch. 3.2), NG_{EF} admits of gravitational wave vacuum solutions for which the Einstein tensor *doesn't* vanish.[156] Such gravitational radiation would possess energy –as one would indeed expect.

Yet, three reasons curb the proposal's appeal: the unclear definition of *total* energy-stress, the violation of energy conditions, and its limit.

The first problem arises from the fact that in NG_{EF} (in contrast to GR) $\mathbf{G}$ isn't directly connected to the energy-stress tensor of matter; via its field equation (16), only their respective traces

[154] A second problem with pseudotensors carries over from GR: There exists infinitely many pseudotensors, not all of them physically equivalent (ibid.).

[155] This proposal was suggested in the context of GR by Lorentz and Levi-Civita (see Pauli, 1981, fn. 350-351; Cattani & De Maria, 1993, esp. Ch. 5-13).

[156] Introduce the double null coordinates $u = r - t$ and $v = r + t$ for Minkowski spacetime in polar coordinates. Then, the line element for Minkowski spacetime takes the form (see e.g. Straumann, 2012, Ch. 4.8.1):

$$ds^2 = -dudv + \frac{1}{4}(v-u)^2(d\vartheta^2 + \sin^2\varphi).$$

For an *arbitrary* smooth function $f(u)$, we thus construct a family of exact vacuum solution for NG_{EF} from the line-element

$$ds^2 = \exp\big(f(u)\big)\,(2dudv + dx^2 + dy^2).$$

This describes a plane gravitational wave in NG_{EF} of generic amplitude, travelling in the z-direction.

are. It's therefore unclear how to define *total* (gravitational cum non-gravitational) energy-stress. Even if one restricts oneself to only a linear combination, i.e. $\mathbf{T}_{(tot)} \coloneqq \alpha\mathbf{G} + \mathbf{T}$ for some $\alpha \in \mathbb{R}$, (say, in order to preserve the covariant conservation law, ${}^{(g)}\nabla \cdot \mathbf{T}_{(tot)} = \mathbf{0}$), α remains undetermined.

A second problem stems from the lack of algebraic constraints on the *trace-free* Ricci tensor, $\boldsymbol{S} \coloneqq \mathbf{Ric} - \frac{1}{4}R\boldsymbol{g}$. Recall the decomposition of the Riemann tensor into three pieces, built from the Ricci scalar, the trace-free Ricci tensor and the Weyl tensor (see e.g. Wald, 1984, Ch. 3.2), respectively. Via eq. (14), NG_{EF} fixes the Weyl tensor. Via eq. 16, it algebraically relates the Ricci scalar and the trace of the matter energy-stress. But $\boldsymbol{S}$ is left algebraically unconstrained. Via the Bianchi identity and eq. (16), it remains only *differentially* related to matter energy-stress: $\nabla \cdot \mathbf{S} = -6\pi{}^{(g)}\nabla T \otimes \boldsymbol{g}$.

It's therefore possible to make the gravitational energy-density $\mathbf{G}(\boldsymbol{\xi},\boldsymbol{\xi}) \equiv \mathbf{S}(\boldsymbol{\xi},\boldsymbol{\xi}) - 6\pi T\mathbf{g}(\boldsymbol{\xi},\boldsymbol{\xi})$, associated with a time-like observer $\boldsymbol{\xi}$, violate all energy-conditions (see e.g. Malament, 2012, Ch.2.5 for more on energy conditions). Consider, for instance, electro-vacuum, for which $T = 0$. The gravitational energy-stress density for $\boldsymbol{\xi}$ then simplifies to $\mathbf{G}(\boldsymbol{\xi},\boldsymbol{\xi}) = \mathbf{S}(\boldsymbol{\xi},\boldsymbol{\xi})$. Here, $\mathbf{S}$ can be an *arbitrary*, symmetric rank-2 tensor field satisfying ${}^{(g)}\nabla \cdot \mathbf{S} = 0$. In particular, we can choose $\mathbf{S}$ such that it violates the dominant energy condition. The latter states the causal flux of the gravitational energy-stress – a plausible desideratum for any physical energy-stress. This subverts the suitability of $\mathbf{G}(\boldsymbol{\xi},\boldsymbol{\xi})$ as a physical energy-stress density.

A final obstacle to interpreting the Einstein tensor as gravitational energy-stress is posed by its limit. For weak fields, we get in linear order in Ω^2 for the vacuum case:

$$2R_{ab}[\Omega^2\eta] \approx \partial_{a,b}\Omega^2 = 2\partial_a\Omega\partial_b\Omega + 2\Omega\partial_{a,b}\Omega. \tag{25}$$

In this limit, we would expect a reasonable candidate for NG_{EF}'s gravitational energy-stress to recover the Newtonian gravitational energy density. The preceding expression clearly doesn't.

In conclusion, NG_{EF}'s Einstein tensor is no viable candidate for NG_{EF}'s gravitational energy. Neither was NG_N's canonical energy-stress for a naïve Lagrangian approach. Pseudotensors turned out to be viciously coordinate-dependent, gauge-quantities. Gravitational energy thus looks like a tenuous notion in NG_{EF}.

In short, the second mystery of NG, (M2), is this: whereas NG_N admits of a standard energy-stress associated with its scalar, in NG_{EF} gravitational energy-stress ceases to be meaningful in any obvious sense. This predicament is disturbing: isn't it an objective matter of fact −rather than dependant on the choice of two versions of a theory− whether gravity can be attributed energy or not?

VI.3.3. Third Mystery: Energy conservation

Interlocked with the status of gravitational energy is that of energy conservation. In NG_N, *total* energy-stress appears to be conserved; in NG_{EF}, only *non-gravitational* matter energy-stress does.

In NG_N, the continuity equation $\boldsymbol{\nabla} \cdot (\boldsymbol{\Theta} + \mathbf{T}) = 0$ holds (see §3.2). With $\boldsymbol{\Theta}$ being interpreted as gravitational energy-stress, it's naturally construed as conservation of total energy: the sum of gravitational ($\boldsymbol{\Theta}$) and non-gravitational ($\mathbf{T}$) energy-stress is locally (differentially) conserved. The density/flux of *total* energy-stress, $\boldsymbol{\Theta} + \mathbf{T}$, has neither sinks nor sources.

Also globally (integrally), energy is conserved. Recall that Minkowski spacetime has ten Killing fields $\boldsymbol{\zeta}$ (defined via the metric's vanishing Lie-drag along them, $\mathcal{L}_{\zeta}\boldsymbol{g} = 0$) – the generators of the inhomogeneous Poincaré group. Minkowski spacetime's time-like Killing field $\boldsymbol{\xi}$ – the generator of time-translations – underwrites a global conservation law (see e.g. Padmanabhan, 2010, Ch. 6.5) for the total energy-flux $\boldsymbol{J}[\boldsymbol{\xi}] \coloneqq (\boldsymbol{\Theta} + \mathbf{T}) \cdot \boldsymbol{\xi}$ in the direction of $\boldsymbol{\xi}$, passing through the Cauchy hypersurface Σ (with the directed 3-volume element $d\boldsymbol{\Sigma}$):

$$E_{tot}[\boldsymbol{\xi}] \coloneqq \int_{\Sigma} d\boldsymbol{\Sigma} \cdot \boldsymbol{J}. \tag{26}$$

Thanks to $\boldsymbol{\xi}$'s Killing property, the integral on the r.h.s. doesn't depend on the choice of Σ. We may thus interpret $E_{tot}[\boldsymbol{\xi}]$ as the total energy-stress, stored in the time-slice Σ. Also across time, it's conserved. For a (3+1) foliation, $\{\langle \sigma, \Sigma_{\sigma} \rangle\}$, E_{tot} remains constant:

$$\frac{d}{d\sigma} \int_{\Sigma_{\sigma}} d\boldsymbol{\Sigma} \cdot \boldsymbol{J} = \boldsymbol{0}. \tag{27}$$

NG_N thus admits of both a satisfactory local and global energy conservation law. This result chimes with the predominant verdict in the extant literature (e.g. Giulini, 2008; Deruelle, 2011; Deruelle & Sasaki, 2011; Gourgoulhon, 2012). It also meshes with the decisive role that energy conservation played for Einstein as a physical heuristic (Norton, 1993, 2005, 2018; Pitts, 2016a).

Again, in NG_{EF} complications surge. In the preceding subsection, we already described one impediment: in NG_{EF}, the concept of gravitational energy comes under assault. A kindred problem is this: NG_N's continuity equation for total energy-stress, is superseded in NG_{EF} by ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$. How to interpret it? Mirroring the situation in GR, one has three options: it codifies 1. energy *transfer* between gravity and matter, 2. generic energy *non*-conservation or 3. conservation, respectively.

Following Einstein in his 1916 review of GR (see Hoefer, 2000, p.191), some authors (e.g. Weinberg, 1972, p. 166; Brading & Brown, 2002, p. 17) view ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ as expressing the energy exchange between matter and gravity. Several difficulties militate against this interpretation (Dürr, 2018a, §2.1). (The arguments carry over verbatim to NG.) I'll not further pursue it here.

Other authors (e.g. Padmanabhan, 2010, p. 213) espouse a weaker interpretation. According to them, it expresses the *breakdown* of (non-gravitational) matter energy conservation in *generic*, non-inertial reference frames. Contrariwise, in *inertial* frames, matter energy-stress is conserved: owing to the Equivalence Principle, in them, gravitational energy contributions are eliminated.

A third group of authors (e.g. Eddington, 1923, Ch. 59; Dürr, 2018ab) further relax the interpretation of ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$: according to them, it expresses energy *conservation* of (non-gravitational) matter alone. The apparent violation of energy conservation in non-inertial frames is discarded as an artefact of non-adapted, unphysical coordinates, comparable to the appearance of fictitious forces in classical mechanics.

On either interpretation, the status of ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ as conservation law in NG_{EF} appears to be in tension with the *total* energy conservation law in NG_N. In NG_{EF}'s inertial frames (in which $\left\{ {\lambda \atop \mu\nu} \right\}_{\boldsymbol{g}} = 0$, cf., for instance, Knox, 2013, p. 349), only energy-stress of *non*-gravitational matter is conserved, $\partial_\nu T_\mu^\nu = 0$.

In short, NG's third mystery, (M3) is this: total (gravitational *cum* non-gravitational) energy-stress is conserved in NG_N; in NG_{EF}, by contrast, only energy-stress of non-gravitational matter alone is (at least in inertial frames).

(M3) is paradoxical, insofar as both the split between gravity and non-gravitational matter and the distinction between inertial and non-inertial frames plausibly reflect objective distinctions (i.e. grounded in facts, independent of the version of NG one adopts), rather than being merely conventional stipulations: given these assumptions, one would expect the conservation of total energy to be a fundamental principle whose validity doesn't depend on whether one adopts Nordström's or Einstein and Fokker's version of NG.

In conclusion: Depending on whether we consider NG_N or NG_{EF} we receive different answers to three foundational questions about objective matters of fact:

> (M1) Do particles in free-fall follow spacetime geodesics? In NG_N, it appears, they don't; in NG_{EF}, it appears, they do.
> (M2) Is gravitational energy a robustly meaningful physical quantity? For NG_N, one can affirm this; in NG_{EF}, one is inclined to negate it.
> (M3) Is total energy conserved? For NG_N, the answer seems yes; for NG_{EF}, only non-gravitational energy appears conserved.

VI.4. Denouements

This section will dispel the paradoxes that the Three Mysteries (M1)-(M3) present. To this end, I'll scrutinise three of their central suppositions: realism about spacetime structure posited in NG_{EF} and NG_N; the correctness of their respective interpretations, and their empirical equivalence. I'll first probe the denial of realism about spacetime (§4.1). I'll then critically examine NG_{EF}'s naïve spacetime interpretation (§4.2). Its refinement will enable us to fathom more judiciously the relation between NG_{EF} and NG_N (§4.3). In particular, I'll advance an overarching Weyl-geometric spacetime interpretation of the formalism of both. This interpretation will help us to resolve the Three Mysteries (§4.4).

VI.4.1. Conventionalism

Here, I'll inspect the simplest strategy to defang the paradoxes (M1)-(M3): to embrace conventionalism about geometry.

The paradoxical effect of NG's Three Mysteries is predicated on realism about NG_N and NG_{EF}: only if one considers spacetime a real physical entity, do the different answers that NG_N and NG_{EF} give to our questions *genuinely* conflict.

I'll bracket generic anti-realism about scientific theories. More germane to the present context is a specific form of anti-realism: conventionalism about spacetime geometry, as endorsed by Poincaré, Reichenbach or Grünbaum (see e.g. Ben-Menahem, 2006 for a comprehensive review). On this view, spacetime structure is a conventional matter of descriptive expediency, rather than a matter of physical fact. For spacetime conventionalists, the mysteriousness of (M1)-(M3) evaporates:

- Ad (M1): The GP explicitly refers to the spacetime's geodesic structure. If the latter is merely conventional, the GP's possible violation for different conventional choices becomes harmless: conventionalism demotes the GP to a convention-relative principle, devoid of absolute content.
- Ad (M2): Energy-stress in general is always defined relative to some (priorly identified) spacetime structure (cf. Lehmkuhl, 2011; Dewar & Weatherall, 2018; Duerr & Read, 2019). Formally this is manifest in the variational formulation, given in §4.2. (Note also that in order to define global (integral) notions of energy-stress −e.g. Noether charges, rather than Noether currents− one integrates local energy-stress over a volume element. Such a volume element again is a spatiotemporal (typically: metric) posit.) Hence, insofar as we deem spacetime structure conventional, we ought to deem energy-stress conventional, as well.
- Ad (M3): The spacetime-relativity of energy mentioned before also impinges upon total energy-stress. Furthermore, the contemplated two interpretations of the vanishing covariant divergence of NG_{EF}'s energy-stress tensor made crucial reference to inertial frames. The conventionality of spacetime, and of particular inertial structure, would thus bleed into the conventionality of one's account of total energy conservation.

Conventionalism about geometry consequently cuts the Gordian knot. However, it has incurred incisive criticism (see e.g. Torretti, 1983, Ch. 7.2; Friedman, 1983; Ch. VII, McKie, 1988; Nerlich, 1994; cf. however Pitts, 2016b). I'll not jump into the fray. Instead, I'll just assume that spacetime realism can be defended.[157] My task then will be to show that NG's

[157] §4.2 will establish that from the perspective of Weyl geometry, NG_N and NG_{EF} pick out merely different gauges of the same Weyl geometric theory. We can then invoke covariance principles to sever the factual and conventional components of each (see e.g. Norton, 1994): the former are identified with what remains invariant under the theory's symmetry transformations. What doesn't remain invariant is identified as the conventional

Mysteries (M1)-(M3) pose no serious threat to spacetime realism[158]: rather, I'll debunk them as artefacts of essentially misidentifying NG_N's *true* spacetime.[159]

VI.4.2. NG_{EF} revisited

Here, I'll revisit NG_{EF}'s spacetime interpretation. I'll present an alternative formulation which cures the former's blemishes. This will pave the way for our subsequent (§4.3) re-considerations of the equivalence between NG_N and NG_{EF}.

All is not well with NG_{EF}. With reason, one may demur to four related features: the absence of a Lagrangian formulation[160], the theory's limitation to test matter, the unclear definition of the theory's energy-stress tensor, and the apparently ad-hoc status of the conservation law (19).

Without a Lagrangian formulation, the metric $\boldsymbol{g}$'s coupling to matter other than test particles remains indeterminate: how does, say, a scalar field propagate on NG_{EF}'s spacetime? This drastically curtails the theory's scope.

From a modern perspective it's desirable to define the energy-stress tensor variationally as $\mathbf{T} := -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta \boldsymbol{g}} S_m$, with the matter action $S_m = \int d^4 \sqrt{|g|} L_m$. Absent a *full* Lagrangian picture, this avenue is obstructed. Unless the theory as whole − i.e. both its gravitational and non-gravitational sector − is amenable to a Lagrangian formulation, the variational formulation of the energy-stress for test matter looks contrived.

NG_{EF}'s non-Lagrangian presentation also obscures the status of ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$: Is it an independent postulate or a theorem? In standard field theory, such conservation laws are

part of the theory's *representational* surplus structure. (The latter is what I'll refer to in the present context as gauge structure.)
Within *Riemannian* geometry, though, this strategy doesn't succeed in warding off conventionalism. Although NG_N and NG_{EF} share the light-cone structure, they differ over their geodesic structure: spacetimes with ${}^{((1+\phi)^2\boldsymbol{\eta})}\boldsymbol{\nabla}$ and ${}^{(\boldsymbol{\eta})}\boldsymbol{\nabla}$ disagree over which time-like curves can be reparameterised as geodesics.

[158] Needless to say, this strategy won't sway the conventionalist. She rejects the idea of super-empirical virtues as guides to *truth*. Any talk of truth, according to conventionalism is misguided: conventions, devoid of factual content, lack truth (see e.g. Ben-Menahem, 2006, Ch. 1&2).

[159] My strategy to bolster this claim closely follows Knox's (2011).

[160] The absence of a satisfactory Lagrangian treatment is endemic to the historical material. Not even in his later work, did Einstein consistently adopt a thoroughgoing Lagrangian strategy for the matter sector (Pitts, 2016a). Plausibly, this is linked to his leeriness about the status of contemporaneous matter theories tout court (cf. Lehmkuhl, 2018).

usually *derived* within a Noetherian framework, i.e. follow from symmetries of the action. But again, here, such reasoning isn't applicable in any obvious sense.

Fortunately, those blemishes can be cured. I'll now present a Lagrangian formulation of NG_{EF}. Begin with NG_{EF}'s KPMs. In §2.2, Weyl-flatness was imposed at the level of NG_{EF}'s DPMs. Here, let's incorporate it already at the level of the *KPMs*, instead. That is: NG_{EF}'s KPMs, I stipulate, are now given by the quadruple

$$\langle \mathcal{M}, (1+\phi)^2\boldsymbol{\eta}, {}^{((1+\phi)^2\boldsymbol{\eta})}\boldsymbol{\nabla}, \boldsymbol{\Psi}\rangle. \qquad (28)$$

Here, $1+\phi$ is a smooth, non-zero scalar. It represents NG_{EF}'s gravitational degrees of freedom, ingrained in the metric's conformal factor.[161] All other denotations carry over from §2.2.

This is both a kosher and meaningful move: KPMs delimit "[…] the range of metaphysical possibilities consistent with the theory's basic ontological assumptions. The DPMs represent a narrower set of physical possibilities" (Pooley, 2013, p. 532). The proposed transition to modified KPMs thus merely recognises Weyl-flatness as *metaphysically* necessary[162] within NG_{EF}, whereas previously it was held only *physically* necessary.

Now consider the total (gravitational *cum* matter) Einstein-Hilbert action for $\boldsymbol{g} \coloneqq (1+\phi)^2\boldsymbol{\eta}$ and the standard matter Lagrangian $L_m(g, \Psi, \partial\Psi, \dots)$: [163]

$$S_{tot}[\boldsymbol{g}, \boldsymbol{\Psi}] = \int d^4\sqrt{|g|}\left(\frac{1}{2\kappa}R + L_m(\boldsymbol{g}, \boldsymbol{\Psi}, \partial\boldsymbol{\Psi}, \dots)\right). \qquad (29)$$

Remarkably[164], its variation with respect to both $\boldsymbol{g}$ and $\boldsymbol{\Psi}$ yields NG_{EF}'s field equation (16), as well as the equations of motion for matter (Romero, Fonseca-Neto & Pucheu, 2012b, sect. 5,

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[161] A (pseudo-)Riemannian metric g_{ab} can be decomposed into two irreducible parts (see e.g. Stachel, 2011), the conformally invariant part $\gamma_{ab} = |g|^{-\frac{1}{4}}g_{ab}$ (a rank-2 tensor density of weight -½), and the conformal factor $V \coloneqq |g|^{\frac{1}{2}}$ (a weight-1 scalar density): $g_{ab} \equiv V^{\frac{1}{2}}\gamma_{ab}$. The former defines the metric's conformal/light-cone structure; the latter defines a volume element, which in turn gives a preferred affine parameter for the metric's geodesics.
[162] This underlines the great importance that, to my mind, constraints a the level of KPMs have for physical theorising (see Curiel, 2016): in particular, they serve as preconditions for the well-definedness of the theory's dynamics (equations of motion), and thereby the theory's applicability.
[163] For technical details, in particular the incorporation of the boundary term, see Poisson (2004), Ch.4.1.
[164] From this (glaringly *ahistorical*!) vantage point, this formulation of NG_{EF} is tantalisingly close to GR: solely NG_{EF}'s *a priori* restriction to conformally flat spacetimes separates the two theories. Lifting this assumption, one recovers standard GR!

6)! (The crucial realisation is that $\delta\boldsymbol{g} = 2(1+\phi)^{-1}\delta\phi\, \boldsymbol{g}.$). As before, they pick out NG_{EF}'s DPMs.

The energy-stress tensor that figures on the r.h.s. of the field eq. (16) can now be identified as the standard one, defined variationally: $\mathbf{T} := -\frac{2}{\sqrt{|g|}}\frac{\delta}{\delta \boldsymbol{g}}\int d^4\sqrt{|g|}\int d^4\sqrt{|g|}\, L_m$. Its vanishing covariant derivative, ${}^{(g)}\boldsymbol{\nabla}\cdot\mathbf{T} = \mathbf{0}$ (which for test-matter coincides with the equations of motion, see e.g. Weatherall, 2018) follows from the general covariance of the matter action, and the form of the Lagrangian in the same manner as in GR (e.g. Hobson, Efstathiou & Lasenby, 2006, Ch. 19.12).

As a spin-off, this Lagrangian formulation affords a natural extension of NG_{EF}'s matter sector beyond its historical restriction to test particles: The familiar minimal coupling scheme can be applied to NG_{EF} no worse than to GR (see e.g. Carroll, 2004, pp, 152, pp. 179).

In conclusion: Applying Hamilton's Principle to the full Einstein-Hilbert action, with the metric constrained to be conformally flat ab initio, yields NG_{EF}'s dynamics. This formulation allows the standard definition of NG_{EF}'s matter energy-stress tensor. Furthermore, it allows NG_{EF}'s natural extension to arbitrary types of (non-quantum) matter. The conservation law now holds as a theorem.

On the basis of this re-formulation of NG_{EF}, we'll next revisit the equivalence between NG_{EF} and NG_N.

VI.4.3. The Equivalence of NG_{EF} and NG_N

This subsection seeks a more circumspect exposition of the nature of the relation between NG_N and NG_{EF}. Let's first (§4.3.1) look into possible challenges for both their theoretical and even empirical equivalence. In §4.3.2, we'll see that for conformal matter both theories can be identified as representational variants of the same theory. Their theoretical equivalence can be extended to generic matter from the (anachronistc) perspective of Weyl geometry, i.e. by embedding them into a Weyl-geometric "super-theory" (§4.3.3).

VI.4.3.1 Challenges for equivalence

Suppose a spacetime realist (in the sense of §4.1) subscribes to the respective interpretations of NG_{EF} and NG_N. By impugning their equivalence, she can still shrug off NG's Three Mysteries:

notwithstanding some structural and empirical overlap, NG_{EF} and NG_N aren't merely representational variants of each other. It then oughtn't faze us, if they disagree on the status and validity of fundamental concepts and principles: they are *distinct* theories.

Two reasons seem to buttress such a stance: scepticism about the empirical equivalence between NG_{EF} and NG_N, and doubts about (or even accounts that dispute) their theoretical equivalence, i.e. theory identity.

First, NG's historical limitation of the matter sector to test particles leaves open the possibility that *other* forms of matter break the empirical equivalence between NG_{EF} and NG_N. With empirical equivalence being a necessary criterion for theory equivalence, NG_{EF} and NG_N's theoretical inequivalence follows trivially.

Secondly, let's concede that NG_{EF} and NG_N share some mathematical similarities. Let's even grant their (permanent/non-transient) empirical equivalence. Yet, one may wish to resist their identification on two grounds. One is that such an identification crucially hinge on details that the discussion so-far has elided: how *exactly* are NG_N and NG_{EF} structurally related? Is, for instance, the mapping between NG_{EF}'s and NG_N 's DPMs one-to-one or one-to-many? A second reason for not identifying NG_{EF} and NG_N may be caution: absent any consensus on *sufficient* criteria for theory individuation/equivalence (see e.g. Weatherall, 2019 for a recent review), it may be prudent to refrain from a judgement of equivalence in the case at hand.

I'll now address the first concern. To that end, I'll next lay bare NG_N and NG_{EF}'s structural relationship in a manner that guarantees their empirical equivalence. This clarification will enable a more nuanced assessment of reasons to also theoretically identify them. I'll flesh out their structural equivalence (i.e. the *mathematical* equivalence of their dynamics– omitting pro tempore questions of *interpretative* equivalence) in two ways. The first centres on the inter-translatability of NG_N and NG_{EF}'s constitutive equations via so-called conformal re-descriptions. The second approach subsumes NG_N and NG_{EF} under an overarching theory: the two theories then come out as equivalent descriptions in terms of two gauges of the same Weyl geometric theory.

VI.4.3.2 The perspective from conformal frames

The perspective from conformal frames is adumbrated in Einstein and Fokker's paper, rehashed in §2.2. NG_N's and NG_{EF}'s dynamics are re-descriptions of each other through a change of units of length and time.

To elaborate, recall NG_{EF}'s field equation

$$R[\boldsymbol{g}] = -24\pi T[\boldsymbol{g}, \boldsymbol{\Psi}]. \tag{30}$$

Thanks to the metric's conformal flatness, the l.h.s. simplifies to an expression for the metric's non-absolute/dynamical degree of freedom, i.e. its conformal factor $1 + \phi = \sqrt[8]{|g|/|\eta|}$:

$$R[(1+\phi)^2\boldsymbol{\eta}] = -6(1+\phi)^{-3}\Box\phi. \tag{31}$$

Let's next evaluate the r.h.s. of eq. (31) in terms of its so-called conformal transform (see e.g. Wald, 1984, Appendix D; Dabrowski, Garecki & Blaschke, 2009 for details). By that the following is meant.

Consider the (non-gravitational) physics unfolding on the generic spacetime $\langle \mathcal{M}, \boldsymbol{g}, {}^{(g)}\boldsymbol{\nabla} \rangle$. Without altering the physical phenomena, re-scale −i.e. stretch and shrink− all distances (whilst preserving all angles). The resulting spacetime then is $\langle \mathcal{M}, \overline{\boldsymbol{g}}, {}^{(\overline{g})}\boldsymbol{\nabla} \rangle$, where $\overline{\boldsymbol{g}} = \Omega^2\boldsymbol{g}$ for a non-vanishing smooth $\Omega: \mathcal{M} \to \mathbb{R}^{\neq 0}$. Such re-scalings are called conformal transformations. Under them, the laws of physics describing the (unaltered) phenomena take of course a different form: one only distorts length and time units. The modified re-description is called a representation in a different conformal frame.

Under conformal transformations, $\boldsymbol{g} \to \overline{\boldsymbol{g}} = \Omega^2\boldsymbol{g}$, energy-stress tensors for classically considered matter − electromagnetism and perfect fluids with pressure (upon suitable redefinitions of pressure and density, which we may set aside here) − transform as $\boldsymbol{T}[\boldsymbol{g}, \Psi] \to \overline{\boldsymbol{T}}[\overline{\boldsymbol{g}}, \boldsymbol{\Psi}] = \Omega^{-2}\boldsymbol{T}[\boldsymbol{g}, \boldsymbol{\Psi}]$ (see e.g. Capozziello & Faraoni, 2011, Ch. 2). Consequently,

$$\overline{T} = tr_{\overline{g}}\{\overline{\boldsymbol{T}}\} = \Omega^{-4}T. \tag{32}$$

Now think of NG_{EF}'s metric $\boldsymbol{g}$ as the conformal transform of $\boldsymbol{\eta}$ with the conformal factor $\Omega = 1 + \phi$. Using eq. (32), we can thus express the r.h.s. of eq. (32) as

$$T = (1+\phi)^{-4}\hat{T}, \tag{33}$$

where $\hat{T}[\eta, \Psi]$ is the conformal transform of the trace of the energy-stress tensor $T[\boldsymbol{g}, \Psi]$.

Combing both expressions (33) and (31), we obtain from for NG_{EF}'s field equation:

$$\Box\phi = -4\pi(1+\phi)^{-1}\hat{T}. \tag{34}$$

This is, of course, the field equation (2) for NG_N's scalar.

By the same token, under conformal transformations, the vanishing covariant divergence of the energy-stress, ${}^{(g)}\boldsymbol{\nabla}\cdot\boldsymbol{T}=\mathbf{0}$, becomes:

$$^{(\bar{g})}\boldsymbol{\nabla}\cdot\overline{\boldsymbol{T}} = -\overline{T}\,^{(\bar{g})}\boldsymbol{\nabla}\, ln\,\Omega \tag{35}$$

Choosing again a conformal frame in which NG_{EF}'s metric is Minkowskian, $\overline{\boldsymbol{g}} = \boldsymbol{\eta}$ (with the Levi-Civita connection $\boldsymbol{\nabla}$), we get**:**

$$\boldsymbol{\nabla}\cdot\widehat{\boldsymbol{T}} = -\hat{T}\boldsymbol{\nabla}\, ln(1+\phi). \tag{36}$$

With eq. (34), this is equivalent to

$$\boldsymbol{\nabla}\cdot\left(\widehat{\boldsymbol{T}}+\boldsymbol{\Theta}\right) = \mathbf{0}, \tag{37}$$

with $\boldsymbol{\Theta} = -\frac{1}{8\pi}\left(\boldsymbol{\nabla}\phi\otimes\boldsymbol{\nabla}\phi - \frac{1}{2}(\boldsymbol{\nabla}\phi)^2\boldsymbol{\eta}\right)$, as in §2.1. This is, of course, NG_N's conservation law.

In conclusion: We can identify NG_N's field equation and conservation law as those of NG_{EF}, if the matter sector is represented in a conformal frame in which $\overline{\boldsymbol{g}} = \boldsymbol{\eta}$**.** (The same holds mutatis mutandis for NG_{EF}'s field equation and conservation law and those of NG_N.)

Via this structural equivalence, we can extend NG_N to electromagnetism and perfect fluids (with pressure), deploying the natural extension of NG_{EF} (§4.2). With NG_N and NG_{EF} thus being merely conformal re-descriptions of the classical matter sector, they share the same empirical content.

Essential to the argument is the identity (33) between the energy-stress and its conformal transform. Unfortunately, for matter other than Maxwellian electromagnetism or perfect fluids it no longer holds: already a scalar field (e.g. Deruelle & Sasaki, 2011, Appendix) would thus beak the empirical equivalence between NG_N and NG_{EF}.[165]

[165] One might be tempted to dismiss matter other than Maxwellian electrodynamics and perfect fluids as unclassical, and hence extraneous to the analysis of a classical field theory. This objection strikes me as ad-hoc: For the empirical inequivalence of two theories it's irrelevant, whether the matter that breaks the equivalence is classical or not. Furthermore, suppose one were to preclude non-classical, *quantum* matter from one's considerations. Even so, for the *classical* (non-quantised) electromagnetic field with a mass term (cf. Pitts, 2011), the conformal identities no longer hold. Massive electromagnetism, too, would thus break the empirical equivalence between NG_{EF} and NG_N.

How to extend their empirical equivalence beyond Maxwellian electromagnetism and perfect fluids? For that, we need to transition to Weyl geometry (see e.g. Eddington, 1923, Ch. VII; Scholz, 2017, 2018; Wheeler, 2018).

VI.4.3.2 The perspective from Weyl geometry

Weyl geometry generalises Riemannian geometry by allowing for a connection other than the Levi-Civita connection, induced by the metric. It relaxes the postulate of metric compatibility in favour of the non-metricity condition

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = \boldsymbol{\sigma} \otimes \boldsymbol{g}. \tag{38}$$

Here, $\boldsymbol{\sigma} \in T^*\mathcal{M}$ is a 1-form on the manifold $\mathcal{M}$. Henceforth, we'll assume that the connection $\boldsymbol{\nabla}$ has no torsion. (That is: Two vectors parallel-transported along one another form a parallelogram.) For given $\boldsymbol{g}$ and $\boldsymbol{\sigma}$, the non-metricity condition then uniquely determines $\boldsymbol{\nabla}$. A Weyl geometry is fully characterised by the triplet $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle$. (However, such a triplet still contains redundancy. We'll see shortly that a Weyl geometry is more perspicuously characterised as an equivalence class of such triplets.)

Metric incompatibility portends a dramatic departure from Riemannian geometry: parallel transport (defined via $\boldsymbol{\nabla}$) alters vectors. Consider the smooth curve $C = C(\lambda)$ in $\mathcal{M}$, and two vectors U and V along C. Then, parallel-transporting them along C from $P_0 = C(\lambda_0) \in \mathcal{M}$ to some arbitrary other point $P = C(\lambda) \in \mathcal{M}$, changes their inner product:

$$g(U(\lambda), V(\lambda)) = g(U(\lambda_0), V(\lambda_0))\, exp\left(\int_{\lambda_0}^{\lambda} \sigma\left(\frac{d}{d\xi}\right) d\xi\right), \tag{39}$$

with $\frac{d}{d\xi}$ denoting the vector tangent to C. In consequence, a vector's length becomes path-dependent: for a closed curve C, the magnifying factor $\exp\left(\oint \sigma\left(\frac{d}{d\xi}\right) d\xi\right) \neq 1$ picks up information about the path. Being correlated with the length of their worldlines, clocks thus depend on their history. In particular, spectral lines transmitted via light that passes through a region of generic $\boldsymbol{\sigma}$ cease to be sharp. This so-called "second-clock effect" doomed Weyl's unification of electromagnetism and gravity, based on Weyl geometry (see e.g. Pais, 1982. Ch. 17 (d)) .[166]

[166] Weyl (1918, 1919) identified $\boldsymbol{\sigma}$ with the electromagnetic 4-potential $\boldsymbol{A}$. Interestingly, Weyl's original motivation was a unified field theory; instead, he sought to "do justice mathematical justice" (Afriat) to lengths and directions by a purely infinitesimal geometry (Ryckman, 2005, Ch. 6; Afriat, 2009).

The second-clock effect can be eschewed, however (e.g. Lobo & Romero, 2018; Romero, Lima & Sanomiya, 2019): if $\boldsymbol{\sigma}$ is exact (i.e. of the form $\boldsymbol{\sigma} = \boldsymbol{\nabla}\varphi$ for some scalar φ), Stokes' theorem ensures

$$\oint \sigma\left(\frac{d}{d\xi}\right) d\xi = 0. \tag{40}$$

The magnifying factor in (39) is thus path-*in*dependent. In the following we'll restrict our considerations to such cases.

But triplets $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle$ with exact $\boldsymbol{\sigma}$ retain another effect peculiar of Weylian geometries. The length $L(\lambda)$ of a vector at $P = C(\lambda)$ changes, when parallel-transported along a (non-closed) curve C. For a coordinate basis $\{x^a\}$, eq. (39) implies

$$\frac{d}{d\lambda} L = \frac{\sigma_a}{2} \frac{dx^a}{d\lambda} L. \tag{41}$$

The characterisation of Weyl geometry so far, via triplets $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle$, contains redundancy. To pare down this surplus structure, notice that the non-metricity condition (38) is invariant under the following simultaneous ("Weyl") transformations:

$$\begin{cases} \boldsymbol{g} \\ \boldsymbol{\sigma} \end{cases} \rightarrow \begin{cases} \widetilde{\boldsymbol{g}} := e^f \boldsymbol{g} \\ \widetilde{\boldsymbol{\sigma}} := \boldsymbol{\sigma} + \boldsymbol{\nabla} f = \boldsymbol{\nabla}(\varphi + f) \end{cases}, \tag{42}$$

for an arbitrary scalar $f: \mathcal{M} \rightarrow \mathbb{R}$.

Weyl transformations thus induce an equivalence relation $\sim$ between triplets: $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle \sim \langle \mathcal{M}, \widetilde{\boldsymbol{g}}, \widetilde{\boldsymbol{\sigma}} \rangle$, if and only if they are related via Weyl transformations. The relation $\sim$ then defines the equivalence class of triplets $[\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle] := \{\langle \mathcal{M}, \widetilde{\boldsymbol{g}}, \widetilde{\boldsymbol{\sigma}} \rangle : \langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle \sim \langle \mathcal{M}, \widetilde{\boldsymbol{g}}, \widetilde{\boldsymbol{\sigma}} \rangle\}$. Such an equivalence class of triplets is called an integrable Weyl geometry. (The qualifier signifies the non-metricity vector is exact. For readability, I'll suppress it: the Weyl geometries considered here will all be integrable.) Different choices for the metric and non-metricity field as representatives of the equivalence class are different mathematical representations of the same Weyl geometry. In other words, Weyl transformations (42) are gauge transformations of the Weyl geometric facts: any pair of triplets $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle$ and $\langle \mathcal{M}, \widetilde{\boldsymbol{g}}, \widetilde{\boldsymbol{\sigma}} \rangle$, related via Weyl transformations represent the same Weyl geometry: $[\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle] = [\langle \mathcal{M}, \widetilde{\boldsymbol{g}}, \widetilde{\boldsymbol{\sigma}} \rangle]$.

What are the salient, invariant structures of a Weyl geometry? It's fully characterised by its geodesic structure, a distinguished metric structure (Romero, Fonseca-Neto & Pucheu, 2012a,

§2). The former follows from the observation that Weyl transformations leave the connection coefficients invariant:

$$\Gamma^a_{bc} = \begin{Bmatrix} a \\ bc \end{Bmatrix}_g - \frac{1}{2} g^{ad}(g_{db}\sigma_c + g_{dc}\sigma_b - g_{bc}\sigma_d) \to \tilde{\Gamma}^a_{bc} = \Gamma^a_{bc}. \quad (43)$$

For the privileged metric structure, realise first that through a suitable Weyl transformation of a triplet $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle$, one can always restore metric compatibility. Consider what I'll henceforth refer to as the "effective metric" $\widehat{\boldsymbol{g}} \coloneqq \widehat{\boldsymbol{g}}[\boldsymbol{g}, \sigma] \coloneqq \exp(-\int \sigma)\, \boldsymbol{g} = \mathrm{e}^{-\varphi} \boldsymbol{g}$. The *non*-metricity condition (38) is equivalent to the (Riemannian) metricity condition for it:

$$\boldsymbol{\nabla} \cdot \widehat{\boldsymbol{g}} = \boldsymbol{0}. \quad (44)$$

$\widehat{\boldsymbol{g}}[\boldsymbol{g}, \sigma]$ doesn't depend on the representative $[g, \sigma]$ of its associated equivalence class. (This justifies the above representation-independent notation $\widehat{\boldsymbol{g}}$). $\widehat{\boldsymbol{g}}$ therefore invariant under Weyl transformations:

$$\widehat{\boldsymbol{g}} \to \widehat{\boldsymbol{g}}. \quad (45)$$

$\widehat{\boldsymbol{g}}$ thus defines a privileged metric structure, preserved under Weyl transformations. As a corollary, all geometric quantities constructed from $\widehat{\boldsymbol{g}}$ are likewise invariant. It turns out that its Levi-Civita connection coincides with connection in the original non-metricity condition, $\hat{\Gamma}^a_{bc} := \begin{Bmatrix} a \\ bc \end{Bmatrix}_{\hat{g}} = \Gamma^a_{bc}$.[167]

In consequence, an integrable Weyl geometry $[\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle]$ uniquely determines, and is uniquely determined by a privileged metric structure $\widehat{\boldsymbol{g}}$. It yields a particularly natural representative of the Weyl geometry − the so-called Riemann gauge:[168]

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[167] Another fundamental invariant of Weyl geometry is what Weyl dubbed length curvature ("Streckenkrümmung"): $\boldsymbol{F} \coloneqq d\boldsymbol{\sigma} = 2\partial_{[a}\sigma_{b]}dx^a \wedge dx^b$. Whereas Riemann curvature is responsible for changes in the direction of parallel-transported vectors (hence dubbed "direction curvature" ("Richtungskrümmung") by Weyl), length curvature regulates their length. For exact $\boldsymbol{\sigma}$, the length curvature vanishes: $\boldsymbol{F} \equiv \boldsymbol{0}$. This reflects the above-mentoined fact that integrable Weyl geometries escape the Second Clock Effect.

[168] Notice that $\hat{g}$ thus plays a double role. First and foremost, the "effective metric" $\hat{g}_{ab}$ denotes a gauge-invariant object: conceived of as functional $\hat{g}_{ab}[g, \sigma]$, it's independent of any representative of the Weyl geometry's equivalence class, $\langle \boldsymbol{g}, \boldsymbol{\sigma} \rangle \in [\langle \boldsymbol{g}, \boldsymbol{\sigma} \rangle]$. (This is analogous to the electromagnetic Faraday tensor $F_{ab}[A] := 2\partial_{[a}A_{b]}$: it, too, is independent of the representative of the equivalence class of electromagnetic 4-potentials, related via gauge transformations (i.e. that differ by an exact 1-form).)
The (gauge-*variant*) gauge-metric of a Weyl geometry (i.e. the representative of the latter's equivalence class) can be made – by going to the Riemann gauge – to *coincide* with it. But this doesn't mean, of course, that the Riemann gauge metric should be *identified* with $\hat{g}_{ab}$. The Riemann gauge metric and $\hat{g}_{ab}$ are distinct objects: the latter is an invariant metric, defined on the integrable Weyl Geometry, the latter is a representative of the equivalence class $[\langle \boldsymbol{g}, \boldsymbol{0} \rangle]$ defining that Weyl Geometry .
I thank Dennis Lehmkuhl (Bonn) for pressing me on this.

$$[\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma} \rangle] = [\langle \mathcal{M}, \widehat{\boldsymbol{g}}, \boldsymbol{0} \rangle]. \tag{46}$$

What is the connection between Weyl geometry and NG? The field equations of $\mathrm{NG_N}$ and $\mathrm{NG_{EF}}$ pick out different gauges (representatives) of the same Weyl geometry (Romero, Fonseca-Neto, 2012b, §5, 6). I'll now expand on this claim.

In §2.2, we portrayed $\mathrm{NG_{EF}}$ as theory about the conformally flat spacetime $\langle \mathcal{M}, \boldsymbol{g}, {}^{(g)}\boldsymbol{\nabla} \rangle$. By construction, the latter exemplifies a Riemannian geometry: ${}^{(g)}\boldsymbol{\nabla} \cdot \boldsymbol{g} = \boldsymbol{0}$. Let's therefore stipulate that $\mathrm{NG_{EF}}$'s field equations determine the Riemann gauge $\langle \mathcal{M}, \widehat{\boldsymbol{g}}, 0 \rangle$ of the Weyl geometry $[\langle \mathcal{M}, \widehat{\boldsymbol{g}}, 0 \rangle]$, with the invariant effective metric $\widehat{\boldsymbol{g}} = \boldsymbol{g} = (1+\phi)^2 \boldsymbol{\eta}$. The corresponding gauge in which the metric is Minkowskian can then be gleaned from (42) as $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla} \ln(1+\phi) \rangle$. That is:

$$[\langle \mathcal{M}, (1+\phi)^2 \boldsymbol{\eta}, 0 \rangle] = [\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla}\, ln(1+\phi) \rangle]. \tag{47}$$

$\mathrm{NG_{EF}}$'s dynamics picks out the metric and Levi-Civita connection of the Riemann gauge. As they are invariants of the Weyl geometry $[\langle \mathcal{M}, \widehat{\boldsymbol{g}}, 0 \rangle]$, both sides of $\mathrm{NG_{EF}}$'s field equations (14) &(16) remain invariant under Weyl transformations.

$\mathrm{NG_N}$'s dynamics picks out the scalar $1+\phi$ of the non-metricity field in the gauge $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla} \ln(1+\phi) \rangle$. To procure $\mathrm{NG_N}$'s field equations, we merely have to express $\mathrm{NG_{EF}}$'s field equations in terms of $\boldsymbol{\eta}$ and its Levi-Civita connection $\boldsymbol{\nabla} = {}^{(\eta)}\boldsymbol{\nabla}$.

To that end, observe first that since $\mathrm{NG_{EF}}$'s metric $\boldsymbol{g}$ is assumed to coincide with the invariant $\widehat{\boldsymbol{g}}$, $\mathrm{NG_{EF}}$'s matter action is invariant under Weyl transformations (42):

$$S_m[\boldsymbol{g}, \boldsymbol{\Psi}] = \int d^4x \sqrt{|\widehat{g}|}\, L_m(\widehat{\boldsymbol{g}}, \boldsymbol{\Psi}) \to S_m[\widetilde{\boldsymbol{g}}, \boldsymbol{\Psi}] = S_m[\boldsymbol{g}, \boldsymbol{\Psi}]. \tag{48}$$

The energy-stress tensor with respect to the Riemann gauge metric, i.e. $\boldsymbol{T} \equiv -\frac{2}{\sqrt{|g|}} \frac{\delta}{\delta \boldsymbol{g}} S_m[g, \Psi]$, can therefore be expressed in terms of the energy-stress with respect to the non-Riemann gauge metric $\widetilde{\boldsymbol{g}}$ as:

$$\widetilde{\boldsymbol{T}} \equiv -\frac{2}{\sqrt{|\widetilde{g}|}} \frac{\delta}{\delta \widetilde{\boldsymbol{g}}} S_m[\widetilde{\boldsymbol{g}}, \boldsymbol{\Psi}] = e^f \boldsymbol{T}. \tag{49}$$

So, with $f = 2\ln(1+\phi)$ the link between $\mathrm{NG_N}$'s energy-stress tensor $\widetilde{\boldsymbol{T}}$ (defined with respect to $\boldsymbol{\eta}$) and $\mathrm{NG_{EF}}$'s energy stress-tensor (defined with respect to $\boldsymbol{g}$) is:

$$\widetilde{\boldsymbol{T}} = (1+\phi)^2 \boldsymbol{T}. \tag{50}$$

For the respective traces, it follows that

$$\tilde{T} := tr_{\boldsymbol{\eta}}\{\tilde{\boldsymbol{T}}\} = (1+\phi)^4 T \tag{51}$$

With this identity, we have recovered NG_N's field equations (2):

$$\Box\phi = 4\pi(1+\phi)^{-1}\tilde{T}. \tag{52}$$

By the same token, for NG_N's conservation law, we merely need to rewrite the vanishing covariant derivative of the energy-stress tensor in the Riemann gauge $\langle \mathcal{M}, \hat{\boldsymbol{g}}, \boldsymbol{0}\rangle$

$$\boldsymbol{0} = {}^{(\hat{g})}\boldsymbol{\nabla}\cdot\boldsymbol{T} = {}^{(\hat{g})}\boldsymbol{\nabla}\cdot\left(e^{-f}\tilde{\boldsymbol{T}}\right) \tag{53}$$

in terms of the Levi-Civita connection of the metric $\boldsymbol{g}$ in the gauge $\langle \mathcal{M}, \boldsymbol{g}, \boldsymbol{\sigma}\rangle$. This is accomplished via the following relation between the connection (with the components $\hat{\Gamma}^a_{bc}$) and the Levi-Civita connection ${}^{(g)}\boldsymbol{\nabla}$ (with the components $\left\{ {a \atop bc} \right\}_g$) for any gauge of a Weyl geometry $[\langle \mathcal{M}, \hat{\boldsymbol{g}}, 0\rangle]$:

$$\hat{\Gamma}^a_{bc} = \left\{ {a \atop bc} \right\}_g - \frac{1}{2} g^{ad}(g_{db}\sigma_c + g_{dc}\sigma_b - g_{bc}\sigma_d). \tag{54}$$

For the gauge $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla}\ln(1+\phi)\rangle$, we obtain:

$$\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{T}} = -\tilde{T}\boldsymbol{\nabla}\, ln(1+\phi). \tag{55}$$

with $\boldsymbol{\nabla} = {}^{(\eta)}\boldsymbol{\nabla}$.

Together with NG_N's already recovered field equation, this eventually leads to the conservation law $\boldsymbol{\nabla}(\boldsymbol{T}+\boldsymbol{\Theta}) = \boldsymbol{0}$.

The Weyl geometric perspective affords an elegant understanding of the direct and universal coupling of NG_N's scalar ϕ to the test matter variables in §2.1. Above we argued that NG_N's field equation and conservation law determine the gauge $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla}\ln(1+\phi)\rangle$ of the integrable Weyl geometry $[\langle \mathcal{M}, (1+\phi)^2\boldsymbol{\eta}, 0\rangle]$.

In this gauge**,** the non-metricity condition (38) takes the form:

$${}^{(\hat{g})}\boldsymbol{\nabla}\cdot\boldsymbol{\eta} = 2\boldsymbol{\nabla}\, ln(1+\phi)\otimes\boldsymbol{\eta}. \tag{56}$$

As a result, the length L of a parallel transported vector changes according to (41):

$$\frac{dL}{d\lambda} = \frac{\partial_a\phi}{1+\phi}\frac{dx^a}{d\lambda}L. \tag{57}$$

The length of a test-particle's worldline is therefore given by

$$\int \left(|\boldsymbol{\eta}(ds, ds)|^{\frac{1}{2}} exp(2 \int d\, ln(1+\phi)) \right) = \int |(1+\phi)^2 \boldsymbol{\eta}(ds, ds)|^{\frac{1}{2}}. \tag{58}$$

The natural lengths figuring in the laws of physics hence are induced by the effective metric $(1+\phi)^2\boldsymbol{\eta}$.

This discrepancy between NG_N's Minkowskian metric structure and the "naturally measured distances" (and durations) reveals the geometry's (integrable) *Weylian* (rather than Riemannian) nature: it's a manifestation of the non-trivial non-metricity field $\boldsymbol{\sigma} = 2\boldsymbol{\nabla}\ln(1+\phi)$ in the gauge $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla}\ln(1+\phi)\rangle$.

In conclusion: The dynamical equations of NG_N and NG_{EF} select two different gauges (representatives) of the same Weyl geometry $[\langle \mathcal{M}, (1+\phi)^2\boldsymbol{\eta}, 0\rangle]$. They correspond to the Riemann gauge and a non-Riemannian gauge, respectively. Thus construed, NG_N and NG_{EF} are empirically equivalent for all types of matter.

The preceding Weyl-geometric framework illuminates the empirical equivalence between NG_N and NG_{EF}, as well as the mathematical equivalence between their constitutive equations. We are now able to ascertain their theoretical equivalence: should we identify NG_N and NG_{EF} as different representations of the same theory?

VI.4.3.3 The case for theoretical equivalence

I'll now substantiate the claim that NG_N and NG_{EF} are indeed theoretically equivalent. The argument proceeds in three steps. First, I'll animadvert upon NG_N's field interpretation. I'll then show that NG_{EF}'s (refined) spacetime interpretation, applied to NG_N's formalism, evades the flaws of NG_N's field interpretation. I conclude with pleading for identifying NG_N and NG_{EF} as the same theory.

Return to NG_N's field interpretation of (§2.1): NG_N is a special-relativistic theory about a gravitational scalar. Three shortcomings of this interpretation stand out: an intransparent coupling scheme, its deployment of universal forces and its gauge degrees of freedom.

The first complaint about NG_N, if taken as a theory in its own right (i.e. one that isn't derivative of NG_{EF}), excoriates its scalar's universal, direct coupling. Such a feature is familiar from Brans-Dicke theories in the so-called Einstein Frame (see e.g. Faraoni & Capozziello, 2011, Ch. 3). There, however, one may well dismiss the universal, direct coupling as an artefact of the mathematical representation: instead, the *real* metric is the one to which matter couples effectively − the one whose Levi-Civita connection determines the geodesics of test-particles in free fall (cf. Weinstein, 1996). Arguably more baneful is the artificiality of NG_N's coupling. For point-particles, one may still stomach the scalar's coupling to the matter variables in the matter action (recall §2.1) as a brute fact:

$$S_m = \sum_\nu \, m_\nu \int \phi \, d\tau_\nu. \tag{59}$$

It then comes as a surprise that the gravitational scalar *doesn't* couple to the electromagnetic field. Extending NG_N beyond its historical matter sector (§4.3), one can scarcely gainsay the ad-hoc appearance of the coupling. Consider, for instance, a scalar field φ in a potential $V(\varphi)$. The matter Lagrangian depends *quartically* on the scalar (cf. Deruelle & Sasaki, 2011):

$$S_m \propto \int d^4x \sqrt{|\eta|} \left(\frac{1}{2}(1+\phi)^2 (\nabla\varphi)^2 + (1+\phi)^4 V(\varphi) \right). \tag{60}$$

Such an apparently cobbled-together coupling detracts from NG_N's coherence.

Related to the scalar's universal coupling is a second oddity: NG_N exhibits a "universal force" in the sense of Reichenbach (1957, esp. Ch. I.3 and I.5; cf. Carnap, 1966, pp. 169). That is: When transported to a region where the scalar changes by $\Delta\phi$, equilibrated matter configurations serving as rods of length L suffer the same distortions ΔL, irrespective of their composition:

$$\frac{\Delta L}{L} = \frac{\Delta\phi}{1+\phi}. \tag{61}$$

This "universal effect" (Carnap) of NG_N's scalar can't be screened off.

In particular, the (suitably reparameterised) curves $\gamma\colon \mathbb{R} \supset I \to \mathcal{M}$ of test particles in free-fall deviate from geodesics, irrespective of their mass:

$$\boldsymbol{\nabla}_{\dot{\gamma}} \dot{\boldsymbol{\gamma}} = \boldsymbol{\nabla} \, ln(1+\phi). \tag{62}$$

NG_N's scalar mediates the universal acceleration responsible for this deflection.

In consequence, NG_N's Minkowskian inertial structure remains *invariably* hidden (cf. Renn & Schemmel, 2012, Appendix; Pitts, 2018) – at least, as long as one limits oneself to test matter. In particular, ideal rods and clocks survey the effective metric $(1+\phi)^2\boldsymbol{\eta}$ – not just $\boldsymbol{\eta}$: naturally measured length and time units don't coincide with those determined via the Minkowski metric.

Such universal effects and forces are notoriously suspect. Following Reichenbach (cf. also Dieks, 1987; Nerlich, 1994 Ch. 7.6), whenever we encounter them in a theory, we ought to reformulate it, such that in the new formulation all universal effects and forces are set to zero. (I merely regard this as a – defeasible! – methodological maxim. I don't see anything apriori physically absurd or metaphysically incoherent in the idea of universal forces.) Norton (1994, p. 165) likens them to "[…] the fairies at the bottom of my garden. We can never see these fairies when we look for them because they always hide on the other side of the tree. I do not take them seriously exactly because their properties so conveniently conspire to make the fairies undetectable in principle."

Genuine forces are causes: they explain why a certain event (or default behaviour) occurs – rather than another (cf. Maudlin, 2007, Ch. 5). Universal forces don't allow for such contrast classes: as they can't be switched off, one is unable to evaluate the relevant counterfactuals.

The third, and most devastating objection to NG_N reprimands its interpretatively unaccounted for gauge degrees of freedom. NG_N's dynamical symmetry group – the group of transformations that preserve its dynamical equations – is *larger* than the Poincaré group. One can straightforwardly verify that in addition to the Poincaré transformations, it contains dilatations with simultaneous field re-definitions

$$\begin{cases} x^a \\ 1+\phi \end{cases} \rightarrow \begin{cases} x'^a = \lambda x^a \\ 1+\phi' = \lambda^{-1}(1+\phi')' \end{cases} \tag{63}$$

for some parameter $\lambda \in \mathbb{R}^{\neq 0}$, as well as so-called special conformal transformations $x^a \rightarrow x'^a = 2\beta_b x^b x^a - x^2 x^a$ (for non-null $\boldsymbol{\beta}$) with more complicated simultaneous field re-definitions. Together, the three groups form a 15-dimensional group – the so-called "Bateman-Cunningham Conformal Group" (Pitts, 2016b).

NG_N's symmetry under those transformations differs in no obvious way from the freedom to add a total differential $\boldsymbol{\nabla}\Lambda$ to the electromagnetic 4-potentials $\boldsymbol{A} \rightarrow \boldsymbol{A} + \boldsymbol{\nabla}\Lambda$: they don't seem to alter the physical facts. In other words, they are plausibly gauge transformations[169]: two

[169] This follows what Norton (2019, sect. 10.3) calls a "convenient template when […] to decide if something is a gauge freedom or not": transformations under the Bateman-Cunningham Conformal Group don't result in any observable effects; nor are NG_N's laws able to distinguish between models, related by them.
The case for gauge equivalence can be spelt out more formally (in complete analogy to Newton-Cartan Theory), for instance, along the lines of Weatherall's (2016, 2016b, 2019, 2020) proposal for gauge equivalence in terms of categorical equivalence.

DPMs of NG_N $\langle \mathcal{M}, \eta_{\mu\nu}, \begin{Bmatrix} \kappa \\ \mu\nu \end{Bmatrix}_\eta, \phi, \Psi^{\mu_1 \cdots}_{\nu_1 \ldots} \rangle$ and $\langle \mathcal{M}, \eta_{\mu'\nu}, \begin{Bmatrix} \kappa' \\ \mu'\nu' \end{Bmatrix}_\eta, \phi', \Psi^{\mu'_1 \cdots}_{\nu'_1 \ldots} \rangle$, with the respective coordinatisations $\{x^\mu\}$ and $\{x^{\mu'}\}$, represent the same world, if the two models are related via Poincaré transformations, dilatations or special conformal transformations and their respective simultaneous field redefinitions.

Vis-á-vis those gauge degrees of freedom, one may repudiate NG_N along two lines. One is to demand a symmetry-free interpretation of NG_N: what is the gauge-invariant reality NG_N purports to limn? In particular, what is the status of NG_N's scalar? A second line of criticism targets NG_N's spacetime setting more specifically. NG_N's dynamical symmetries −15 degrees of freedom− and the symmetries of Minkowski spacetime − 10 degrees of freedom− don't match. Neither the dilatations nor the special conformal transformations (together with their respective scalar redefinitions) are symmetries of Minkowski spacetime. NG_N consequently breach Earman's (1989, Ch. 3.4) first adequacy condition for spacetime settings: every dynamical symmetry should be a spacetime symmetry. "The theory that fails [this principle, P.D.] is thus using more spacetime-structure than is needed to support the laws, and slicing away this superfluous structure serves to restore [this principle]" (Earman, 1989, pp. 46).[170] The identification of NG_N's spacetime as Minkowski thus seems spurious. .

In conclusion, NG_N's field interpretation is multiply defective. Can we do better? I affirm this in the next step: the (refined) spacetime interpretation, originally proposed for NG_{EF}, delivers a more satisfactory interpretation of NG_N's formalism, as well.

In §4.3.2, NG_N's formalism was shown to pick out the Weyl geometry described by NG_{EF} as its Riemann gauge. A formalism per se is interpretatively neutral. So, let's stipulate NG_N and NG_{EF}'s *interpretative* equivalence:[171] NG_N too describes the Weyl geometric spacetime $[\langle \mathcal{M}, \widehat{\boldsymbol{g}}, \boldsymbol{0} \rangle]$ with the invariant physical metric $\widehat{\boldsymbol{g}} = (1+\phi)^2 \boldsymbol{\eta}$, and the connection $\widehat{\boldsymbol{\nabla}}$, with the components $\Gamma^a_{bc} = \begin{Bmatrix} \mathrm{a} \\ bc \end{Bmatrix}_{\widehat{\boldsymbol{g}}}$. NG_N just picks out a non-Riemannian gauge. Neither NG_N's scalar nor the Minkowski metric on their own are thus physical quantities; they are gauge-quantities:

[170] Adherents of the so-called dynamical approach to spacetime symmetries, even regard this adequacy condition as analytically true (see Acuña, 2016; Myrvold, 2017).

[171] Asserting interpretative equivalence of NG_N and NG_{EF} goes beyond the foregoing claim that their formalisms pick out different gauges of the same Weyl geometry. NG_N's formalism might happen to be amenable to *two* interpretations (one of which the Weyl geometric one); but only one interpretation accurately limns reality.
This would be an instance of the "discriminatory approach" towards dualities, such as the AdS/CFT correspondence (Le Bihan & Read, 2018, §6; see also Coffey, 2014).

only their gauge-invariant *combination* − in $\widehat{\boldsymbol{g}}$ and Γ^a_{bc} − is invested with physical (viz. chronogeometric and inertial) significance.

This spacetime interpretation of NG_N's formalism overcomes all three shortcomings of the naïve field interpretation.[172] We already saw that in the Riemann gauge (i.e. in NG_{EF}'s formalism), which employs only the gauge-invariant geometric quantities $\widehat{\boldsymbol{g}}$ and Γ^a_{bc}, the minimal coupling scheme is no more problematic than in GR. The above matter action for the scalar in a potential, for instance, becomes:

$$\hat{S}_m \propto \int d^4x\sqrt{|\hat{g}|}\left(\tfrac{1}{2}\left(\hat{\nabla}\varphi\right)^2 + V(\varphi)\right). \quad (64)$$

Likewise, the spacetime interpretation extricates the theory from universal forces. Like in GR (cf. Earman & Friedman, 1973; Dieks, 1983; Nerlich, 2007, 2013; Lehmkuhl, 2014), the effects of gravity are absorbed by the inertial structure, encoded by the connection $\widehat{\boldsymbol{\nabla}}$. In particular, test particles traverse geodesics determined by $\widehat{\boldsymbol{\nabla}}$. As a result, for instance, a cloud of dust changes its volume, when moving across the spacetime, but retains its shape. This isn't due to the presence of a force: rather, it's a purely kinematic effect grounded in the non-Minkowskian – and, in fact, non-Riemannian (viz. Weylian) – inertial structure.

Finally, the spacetime interpretation salvages the validity of Earman's adequacy condition: the (Riemannian) manifold $\langle\mathcal{M}, \widehat{\boldsymbol{g}}, \widehat{\boldsymbol{\nabla}}\rangle$ has $C(1,3)$ as its symmetry group − the conformal group, mentioned in §3.2. NG_N's dynamical symmetries −Poincaré transformations, dilatations and special conformal transformations, together with the redefinitions of the scalar− leave $\widehat{\boldsymbol{g}}$ and $\widehat{\boldsymbol{\nabla}}$ invariant. Hence, all dynamical symmetries are symmetries of the spacetime $\langle\mathcal{M}, \widehat{\boldsymbol{g}}, \widehat{\boldsymbol{\nabla}}\rangle$ (and vice versa −as Earman's (1989, Ch. 3.4) second adequacy condition requires): with $\widehat{\boldsymbol{g}}$ and $\widehat{\boldsymbol{\nabla}}$ being its salient invariant geometric structures, $[\langle\mathcal{M}, \widehat{\boldsymbol{g}}, \mathbf{0}\rangle]$ as the (Weyl-geometric) spacetime setting is now fully adequate.

Having established, empirical, structural and interpretative equivalence between NG_N and NG_{EF}, we are now licenced to pronounce them theoretically equivalent (cf. Møller-Nielsen 2017; Møller-Nielsen & Read, 2018; LeBihan & Read, 2018): they are merely representational variants of the *same* Weyl geometric spacetime theory − NG simpliciter. NG provides the

[172] The spacetime interpretation also allows us to sidestep another, primarily semantic challenge to *any* field interpretation of NG_N, analogous to the case in GR: which quantity (if any!) should we identify as the gravitational field (Lehmkuhl, 2008; Rey, 2013) −e.g. the metric, the Riemann tensor, the connection coefficients?

symmetry-free characterisation of the world which the equivalence class of naïve NG_N models, related via Poincaré transformations, dilatations and special conformal transformations and their concomitant scalar redefinition, describes in different gauge-*dependent* quantities.

It's rewarding to briefly dwell on two analogies that illustrate, respectively, the relation between the naïvely interpreted NG_N and NG_{EF}, and the transition to the Weyl geometric super-theory NG.

Consider first the relation between the field interpretation of models of NG_N to the spacetime interpretation of models of NG_{EF}. It exactly mimics the transition from Newtonian Gravity on Galilei spacetime to Newton-Cartan Theory (e.g. Pooley, 2013; Knox, 2011, 2014; Weatherall, 2016, 2017). The former's symmetry group − the so-called Maxwell group − is larger than the standard Galilei group. It also contains so-called dynamical shifts:

$$\begin{cases}\vec{x}\\ \Phi\end{cases} \rightarrow \begin{cases}\vec{x}' = \vec{x} + \vec{d}(t)\\ \Phi' = \Phi - \ddot{\vec{d}} \cdot \vec{x} + f(t)\end{cases}. \tag{65}$$

Here, $\vec{d}$ is an arbitrary (time-dependent) translation vector, and $f(t)$ an arbitrary scalar constant on simultaneity surfaces. Dynamical shifts have their counterpart in NG_N's dilatations and special conformal transformations, together with their respective simultaneous scalar redefinitions. Just as models of NG_N, related via those transformations, are identified as the same spacetime NG_{EF}, models of Newtonian Gravity on Galilei spacetime, related via dynamical shifts, are identified as the same models of Newton-Cartan Theory (e.g. Malament, 2012, Ch. 4): from the latter's vantage point, dynamical shifts are gauge transformations. Newton-Cartan Theory's symmetry-free description is accomplished via geometrising Newtonian gravity: gravitational phenomena aren't the effects of forces; they are manifestations of a non-Minkowskian inertial structure, encoded in the non-flat connection, subject to the (geometrised) Poisson Equation. In the same vein, NG_{EF} (Riemann-)geometrises NG_N: gravitational phenomena are manifestations of the non-Minkowskian inertial and chronogeometric structure, represented by $\hat{\boldsymbol{g}}$ and $\hat{\boldsymbol{\nabla}}$ of the Riemann manifold $\langle \mathcal{M}, \hat{\boldsymbol{g}}, \mathbf{0}\rangle$.

Identifying models of NG_{EF} and NG_N as gauges of the same integrable Weyl geometry $[\langle \mathcal{M}, \hat{\boldsymbol{g}}, \mathbf{0}\rangle]$ was a conceptually distinct operation. It has an analogy in recognising the formulations of elementary quantum mechanics in momentum-space and the position-space as physically equivalent: they are merely representational/notational variants of the same theory.

In conclusion, we attained *full* equivalence between NG_N and NG_{EF}. They are mathematically equivalent representations of the same theory, NG – a theory about the Weyl-geometric spacetime $[\langle \mathcal{M}, (1+\phi)^2\boldsymbol{\eta}, 0\rangle]$. NG's chronogeometric structure is given by $(1+\phi)^2\boldsymbol{\eta}$; its inertial structure by the associated Levi-Civita connection $\boldsymbol{\nabla}^{((1+\phi)^2\boldsymbol{\eta})}$.

VI.4.4 The Three Mysteries Resolved

We can now harvest the fruits of our labour −and debunk the Three Mysteries of NG of §3. They consisted in putatively conflicting verdicts on the status of the Geodesic Principle (GP), gravitational energy and energy conservation that the naïve field interpretation (NG_N) and the naïve spacetime interpretation (NG_{EF}) yielded.

VI.4.4.1: (M1) -The Geodesic Principle

The First Mystery, (M1), consisted in the GP's representation-dependent validity in NG: in NG_N, one would prima facie expect test particles to follow Minkowskian geodesics; in NG_{EF}, one expect them to follow non-Minkowskian ones.

(M1)'s resolution is now patent: the Mystery rests on the fallacious identification of NG_N's spacetime structure as Minkowskian. NG's spacetime metric is the Weyl-geometry's invariant metric $\hat{\boldsymbol{g}} = (1+\phi)^2\boldsymbol{\eta}$; its associated Levi-Civita connection ${}^{(\hat{g})}\boldsymbol{\nabla}$ defines NG's *spacetime* geodesics, i.e. those representing NG's inertial structure. Test particles (including extended ones in suitable limit) follow these geodesics. The GP thus is restored.

The latter refers to the (unambiguous, i.e. representation-independent) *spacetime* geodesics.[173] It's imperative to heed this specification. Else, the GP can be trivially invalidated:

[173] The insistence that the Geodesic Principle refer to the *spacetime* geodesics (rather than any other geodesic structure *mathematically* definable on a manifold) is underscored most clearly by the Ehlers-Schild-Pirani axiomatisation of geometric theories of gravity (see Ehlers, Schild & Pirani, 1972 for details; cf. also Capozziello et al., 2012). Its starting point are trajectories of test particles in free fall. They define a set of preferred curves. This induces a projective structure on the manifold, uniquely defining a class of geodesics up to re-parameterisation (see e.g. Malament, 2012, Ch. 1.9). Upon further specifying the conformal structure (e.g. by considering the propagation of light rays in the geometric-optical limit), this uniquely fixes a geodesic structure. This procedure delivers the *spacetime* geodesics.

for any given (spacetime) geodesic path one can always find *another* metric for which the path *isn't* a geodesic.

The earlier (§3.1) diagnosis that NG_N doesn't respect the Geodesic Principle flouted this caveat: despite NG_N's formal set-up as a theory on Minkowski spacetime, we opposed the identification of ${}^{(\eta)}\boldsymbol{\nabla}$ as the appropriate spacetime geodesic structure.

4.4.2 (M2) – the Status of Gravitational Energy

NG's Second Mystery, (M2), concerned the apparently ambiguous (representation-dependent) status of gravitational energy-stress. In NG_N, one can ascribe canonical energy-stress to the gravitational scalar $\boldsymbol{\Theta}$ – a prima facie impeccable candidate for gravitational energy. In NG_{EF}, the most plausible approaches to gravitational energy were obstructed.

The ambiguity about the status of gravitational energy is easily dispelled: the naïve realism about gravitational energy suggested by NG_N isn't tenable. Above, NG_N was disclosed as the description of the (non-Riemann) gauge $\langle \mathcal{M}, \boldsymbol{\eta}, 2\boldsymbol{\nabla}\ln(1+\phi)\rangle$ of the Weyl-geometric spacetime $[\langle \mathcal{M}, (1+\phi)^2\boldsymbol{\eta}, 0\rangle]$. Hence, we can immediately discard $\boldsymbol{\Theta}$ as a candidate for genuine gravitational energy: not invariant under the spacetime's symmetries (i.e. under the Weyl transformations (42)), it's a gauge quantity.

The remaining options for gravitational energy-stress echo those perused for NG_{EF}.[174] None is satisfactory. The canonical energy-stress associated with NG's gravitational degrees of freedom (viz. the spacetime metric's conformal factor) – that is: the Noether currents associated with the invariance of NG's gravitational action under rigid translations – yields the pseudotensors we already rejected (§3.2): for them to be to be physically well-defined, they would presuppose more spacetime structure than NG warrants. The other (tensorial)

[174] It's worthwhile spelling out the relationship between NG_N's gravitational energy $\boldsymbol{\Theta}$ and NG_{EF}'s pseudotensor. We identified NG_{EF}'s purely gravitational action as the Einstein-Hilbert action $S_{grav}[g] = \int d^4x\sqrt{|g|}R$ for the conformally flat metric $\boldsymbol{g} = (1+\phi)^2\boldsymbol{\eta}$. One easily verifies the following identity:

$$S_{grav}[g] \equiv -6\oint d\boldsymbol{\Sigma}\cdot\big((1+\phi)\boldsymbol{\nabla}\phi\big) + 12\int d^4x\sqrt{|\eta|}\frac{(\boldsymbol{\nabla}\phi)^2}{2},$$

where $d\boldsymbol{\Sigma}$ denotes the outward-pointing surface element. The term on the r.h.s. is proportional to NG_N's purely gravitational action. In other words: NG_N's and NG_{EF}'s gravitational action (modulo proportionality constants) only differ via a surface term.
This implies that NG_{EF}'s (nonsymmetric) pseudotensor (i.e. canonical Noether current for g), when symmetrised via the Belinfante-Rosenfeld procedure, coincides with NG_N's gravitational energy-stress (Leclerc, 2005). In consequence, the canonical, Noetherian road to gravitational energy in NG_{EF} leads to either a physically unsuitable (nonsymmetric) pseudotensor, or a symmetric and tensorial but gauge-variant quantity (viz. NG_N's canonical energy-stress for the scalar).

proposals for NG's gravitational energy carry over verbatim from NG_{EF}. So do the objections to them. In conclusion, gravitational energy no longer seems a bona fide notion within NG's conceptual framework (again fully analogously to the situation in GR).[175]

This tallies with a broader, "[…] important lesson for how to understand energy in geometrized theories. […] There is a deep relationship between the classical notions of energy, work, force, and inertia. Energy is a measure of the ability to do work […]. But in theories in which gravitation is 'geometrized' in the sense that gravitation is understood as an inertial effect in curved spacetime, we should not think of gravitation as a force at all − and so, in particular, it is not the sort of thing that does work. To the contrary, work makes sense only as a measure of the deviation from inertial motion over some distance" (Dewar & Weatherall, 2018, pp. 26; cf. Dürr & Read, 2019, §4).

4.4.3 (M3) – energy conservation

The apparently representation-dependent validity of total energy conservation formed our Third Mystery. In NG_N, only the sum total of gravitational (represented by $\mathbf{\Theta}$) and non-gravitational energy (represented by NG_N's energy-stress tensor $\mathbf{T}$) appeared to be (locally and globally) conserved, $\mathbf{\nabla}\cdot(\mathbf{\Theta}+\mathbf{T})=\mathbf{0}$; in NG_{EF}, only NG_{EF}'s non-gravitational energy-stress tensor is (locally and globally) conserved, $\widehat{\mathbf{\nabla}}\cdot\widehat{\mathbf{T}}=\mathbf{0}$.

The realisation that NG is a Weyl-geometric spacetime theory immediately removes this prima facie ambiguity about energy conservation. NG_N's account of *total* energy conservation must be dismissed as a reification of a triple gauge-artefact. From the Weyl-geometric perspective, all three elements on the l.h.s. of $\mathbf{\nabla}\cdot(\mathbf{\Theta}+\mathbf{T})=\mathbf{0}$ lack physical significance: The flat connection $\mathbf{\nabla}$ doesn't represent NG's inertial structure; $\mathbf{\Theta}$ is a gauge-quantity (§4.4.2); and $\mathbf{T}\equiv -\frac{2}{\sqrt{|\eta|}}\frac{\delta}{\delta\boldsymbol{\eta}}S_m$ is defined with respect to the gauge metric $\boldsymbol{\eta}$. By contrast, the connection $\widehat{\mathbf{\nabla}}\equiv {}^{((1+\phi)^2\boldsymbol{\eta})}\mathbf{\nabla}$ and energy-stress tensor $\widehat{\mathbf{T}}\equiv -\frac{2}{\sqrt{|\hat{g}|}}\frac{\delta}{\delta\hat{\boldsymbol{g}}}S_m$ in $\widehat{\mathbf{\nabla}}\cdot\widehat{\mathbf{T}}=\mathbf{0}$ invoke physically meaningful and intrinsically distinguished quantities: the connection supplying the geodesic structure of NG's spacetime, and the spacetime metric $\hat{\boldsymbol{g}}$, respectively. Following the arguments in §3.3 for interpreting the vanishing covariant divergence of GR's energy-stress tensor, we conclude:

[175] A bonus of this anti-realism about gravitational energy is a neat solution for the negative energy problem besetting already Newtonian Gravity (e.g. Renn & Schemmel, 2012): if gravitational energy isn't a physical fact, a fortiori it can't be a worrying physical fact that gravitational energy is negative.

$\widehat{\nabla} \cdot \widehat{\mathbf{T}} = \mathbf{0}$ should be construed as a conservation law for (non-gravitational) energy-stress, at least in inertial frames; thanks to the spacetime's time-like Killing vector ξ, the energy-momentum flux $\mathbf{J} \coloneqq \widehat{\mathbf{T}} \cdot \boldsymbol{\xi}$ is even conserved covariantly, see Ch. III. (That this conservation law can be deduced from NG's Lagrangian formulation, given in §4.2, in the standard way, strengthens its status as a law proper.)

VI.5. Conclusion

Behoving philosophy, the present chapter arose out of perplexity: the two empirically equivalent, historical versions of Nordström's theory of gravity depict reality in prima facie incommensurate ways – a gravitational field on special-relativistic spacetime on the one hand, vs. a warped spacetime itself on the other. For worlds adequately described by Nordström's theory, this led to opposing judgements about the validity of the weak equivalence principle, the status of gravitational energy, and energy conservation. Prima facie, the two versions in fact look like distinct, albeit empirically equivalent theories. This gives rise to challenges to a realist interpretation.

Our reflections revealed, however, how to avert conventionalism (or anti-realism) about spacetime. It's possible to overcome the ambiguity regarding those three putatively fundamental, physical notions. Both versions of Nordström Gravity can be subsumed under one overarching super-theory: models of each turn out to be merely different gauges of the same so-called integrable Weyl geometry − Nordström Gravity simpliciter. Thus understood, the two variants of NG are notational variants of each other, rather than distinct theories.

Our analysis of Nordström Gravity is of interest for at least three wider-reaching reasons.

- First, it exemplifies one possible strategy for responding to the challenge of interpreting dualities between theories: the two theories −say, the (putatively) empirically equivalent Schrödinger wave mechanics and Heisenberg's matrix mechanics of the 1920s and 1930s (cf. Muller, 1997ab)− can be embedded into a deeper theory.[176] (This is to be contrasted in particular with the "common core"-

[176] The qualifier "deeper" is particularly apt for the example of Nordström Gravity. Not only does the super-theory NG, unify both Nordström's original theory NG_N and the Einstein-Fokker theory NG_{EF}. Due to its extension beyond the historically considered type of matter, NG's solution space is also (in a natural sense) richer than that of NG_N and NG_{EF} (cf. LeBihan & Read, 2018, §8.1).

strategy. The latter stipulates that one's realist commitment should be confined to the structure common to both dual theories.)

- Secondly, and relatedly, Nordström Gravity illustrates an observation made by Norton (2008) in the context of theory underdetermination: examples of *genuinely* worrying theory underdetermination are rare.

  Already at the level of Nordström's theory and the Einstein-Fokker theory, restricted to their historical matter sector, we saw that the former contains superfluous structure −the (observationally undetectable) Minkowski background structure. (In this regard, NG_N may perhaps be compared to Lorentz's ether theory.) That their respective extensions to generic matter that preserve the two theories' empirical equivalence yield merely notational variants of the same theory is further grist to Norton's mills.
- Thirdly, our discussion serves as an effective antidote against the "fetishism of mathematics" – "[…] the tendency to assume that all the mathematical elements introduced in the formalization of a physical theory must necessarily correspond to something meaningful in the physical theory and even more, in the world that the physical theory purports to help us understand" (Stachel, 1993, p. 149).

  In short: One can't distil a theory's ontology from its formalism (cf. Maudlin, 2013). In particular, a theory's spacetime structure can't be read off *trivially* from its mathematical form. To identify its spacetime structure, one must pay attention to the theory's dynamics, in particular to the way candidate spacetime structure actually couples to matter (cf. Brown, 2005; Knox, 2017; Pitts, 2017; Read, 2018).[177]

  Since claims about a theory's ideology, such as the status of energy conservation, are inextricably intertwined with what one identifies as the theory's spacetime structure, an interpretation of any gravitational theory mandates a circumspect analysis of its actual dynamics.

[177] I wish to stress that this lesson should be heeded to *irrespectively* of one's sympathies of either the dynamical or the geometrical approach to spacetime (cf. Read, 2018).

This chapter:

In the fully geometrised formulation of Nordström Gravity, we found vivid demonstrations of how energy conservation works in generic spacetime theories, and how the notion of gravitational energy is affected by geometrisation – chiming with our previous findings in General Relativity.

The next chapter:

We are now in a position to pull together the threads of this dissertation. I'll now give a general evaluation of the initial working hypotheses regarding energy conservation and gravitational energy.

VII. Conclusion

Let‘s now harvest the fruits of our labours. I’ll first (§1) summarise the findings of this dissertation. Subsequently, in §2, I outline directions of future research.

VII.1. Results

Here, I’ll collate the results of the preceding analyses regarding the status of gravitational energy and energy conservation in geometric theories of gravity.

This dissertation set out with the following working hypotheses:

> **(C1)** In “strength-3 geometric theories of gravity“ (Lehmkuhl) – theories in which gravity is reduced to inertial structure – gravitational energy ceases to be a meaningful, well-defined concept. Like the gravitational force, it’s “geometrised away“.
>
> **(C2)** In generic spacetime theories, energy conservation no longer holds. Rather than an apriori, apodictic principle, it’s contingent on the spacetime’s possessing suitable symmetries.

The previous chapters equipped us with more refined formulations of these working hypotheses. The case studies comprising the bulk of this dissertation corroborated these improved versions.

For reasons of logical dependence, let‘s begin with (C2). Following our discussion in Ch. II, we can distinguish between three forms of *non*-conservation:

(A) Local non-conservation, but global conservation

(B) Local conservation, but global non-conservation

(C) Local and global non-conservation

Option (A) corresponds to a situation in which energy – as a global/integral quantity, ascribable to *regions* of spacetime, or spacetime as a whole – is conserved, whilst at the local/differential level of energy fluxes/densities at a given *point* (represented by an energy-stress tensor), energy either has sources/sinks, or ceases to be well-definable at all. That is: Either sinks/sources exist, but globally compensate each other; or such a local description in terms of sinks/sources isn’t even possible, while the total energy content is both well-defined and conserved. In this sense, energy is *dislocated* from one region to another without being continuously transported across space in a manner that would allow one to track its path of propagation. A non-gravitational analogue is the probability current/flux of the non-relativistic Schrödinger Equation $\vec{j} = \frac{\hbar}{2mi}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*)$:[178] it's a gauge-quantity in the sense that the probability flux is only defined up to a divergence-free term (i.e. up to transformations of the form $\vec{j} \to \vec{j} + \vec{\nabla} \times \vec{A}$); at the level of globally/integrally defined charges, however, they give rise to well-defined charges –the temporal change in (otherwise conserved) probability.

Option (C) corresponds to the strongest possible form of non-conservation. Not only is energy conservation locally violated. (That is: Either sources/sinks of energy fluxes/densities exist, or a description in terms of energy fluxes/densities ceases to be well-defined.) Also globally energy isn't conserved anymore: systems can gain/lose energy without there being identifiable sources/sinks responsible for this change. (No coherent story about the "extra"/"missing" energy is forthcoming anymore. Hence, the talk of energy dislocation – which superseded the usual, but logically stronger talk of energy transport in the case of (A) – must be renounced in the case of (C).) A non-gravitational analogue is entropy in classical thermodynamics: locally, it's only defined up to a thermodynamic gauge transformation (and not well-defined in this sense); globally, it typically grows.

Option (B) occupies a middle position between (A) and (C): although generically (non-gravitational) local energy-stress lacked any sources/sinks, it isn't conserved globally. It was found to be the form of non-conservation, most natural for GR – and somewhat unfamiliar from other theories.

The salient points of (B) can be captured as a sharper formulation of (C2):

> **(C2*)** In generic spacetime theories, the validity of conservation of energy-momentum, both locally and globally, must be qualified.
>
> For generic spacetimes, energy conservation in its *local* form holds for the intrinsically privileged – viz. inertial – frames. For symmetric spacetimes the Killing vectors define energy-momentum fluxes that are conserved in all frames. In both the generic and symmtric case, apparent violations are artefacts.

[178] I choose this example – rather than, say, the 4-currents of a Klein-Gordon or a Dirac field – because typically the Schrödinger Equation isn't presented within a Lagrangian setting; rather, it's postulated as an axiom (see e.g. Messiah, 2014, Ch. II). Consequently, the probability flux is only defined *indirectly* (recall the critique of Hoefer in Ch. III.1) – via the continuity equation, entailed by the Schrödinger Equation.

Global energy conservation is contingent on the spacetime's symmetries (Killing vectors): only in such spacetimes is energy-momentum well-defined and conserved. The increase/decrease in all other cases are brute facts that don't call for any further explanation.

This refinement of our initial working hypothesis, i.e. (C2*), was borne out by the case studies presented in Ch. II-VI:

- In GR[179] (Ch. III), generic spacetimes lack symmetries. An unambiguous/well-defined 4-current of energy-stress of matter then exists only in special frames (viz. the inertial ones). This was argued to constitute a perfectly good sense of local conservation: apparent non-conservation in other frames should be demoted to unphysical artefacts, on an equal footing with the centrifugal or Coriolis force in Classical Mechanics. Globally, the energy enclosed in a volume fails to be conserved: the total amount of (non-gravitational) energy-momentum varies across time.

  If, however, the spacetime is symmetric, energy-momentum is well-defined and conserved both locally and globally. In both cases, the conservation law can be given a covariant form, valid in all possible frames.

- In all variants of Newtonian Gravity – on Newtonian spacetime, on Galilei spacetime, on Maxwell-Huygens spacetime, and in the strength-3 geometrised Newton-Cartan-Theory – energy is conserved both locally and globally. As we saw in Ch. V, their respective spacetime settings all have the required background structure – the vector field generating the Newtonian time pseudo-metric.

- Nordström Gravity, in the strength-3 geometric formulation presented in Ch. VI, admits of a well-defined local and global conservation of energy. Its spacetime group, the conformal group (which has the Poincaré group as a subgroup), supplies the prerequisite Killing fields.

- One can easily find further examples by setting standard gravitational theories (e.g. a massive scalar gravity) on a fixed spacetime background (e.g. an FLRW spacetime). Depending on the latter's symmetries, in the resulting, otherwise perfectly ordinary "theory", conservation ceases to hold.

In Ch. II, III and V it was pointed out that due to the specialness of those spacetimes in GR's space of possible spacetimes, and their lack of robustness (under perturbations), the failure of energy conservation should be assumed as the explanatory default: only its conservation

[179] In Dürr (ms*), I show that this result applies to a much larger class of purely metric extensions of GR, viz. f(R) Gravity (which includes GR as a special case).

calls for an explanation – in the form of symmetries. This generalises to other theories: if the theory posits the prerequisite spacetime symmetries at the level of kinematically possible models – i.e. demands spacetimes with certain symmetries as metaphysically necessary for all physically possible worlds, describable by the theory – energy conservation may be assumed as the explanatorily default principle. Otherwise, energy *non*-conservation should be assumed as the explanatorily default principle.

The link between spacetime symmetries and energy conservation, given by (C2*), should be kept separate from geometrisation (of any strength): per se they are unrelated. Energy conservation can fail to hold – in any form (A)-(C) – in both geometric and non-geometric theories; the same is true of both dynamical and non-dynamical/absolute spacetimes. Solely relevant is the possession of spacetime symmetries.

This leads us to (C1) – the link between strength-3 geometrisation of gravity and the status of gravitational energy. Our analyses in chapters II-VI clarified how the two working hypotheses (C1) and (C2) are linked. They afforded an explication and critical evaluation of the two principal arguments for the oft-perceived need to postulate gravitational energy:

- The first invokes energy ascription as a reality criterion: according to this line of reasoning, only what carries energy counts as real.

  The argument is enthymematic, however: it's predicated on the tacit premise that energy ascription is a necessary (and not merely a sufficient) criterion for the reality of a physical entity.[180] No explicit argument is proffered for this premise in the literature.[181] The standard

[180] For instance, Norton (2019, sect. 4) concludes from the fact that GR's metric "also represents the gravitational field" that "*(t)herefore* it also carries energy and momentum […]" (my emphasis).

[181] An exception is Bunge (2000, 2008). For him, energy precisifies and renders quantitative mutability, the capacity for change. The latter he proposes as a criterion for an entity's materiality/physicality. Endorsing a thorough-going materialism, Bunge only ascribes reality only material/physical entities: he equates physicality/materiality and reality.

Even if granting materialism, I remain sceptical. I'd like to hear more about the connection of mutability and energy – rather than, say, entropy. In particular, it's unclear to me why any dynamical variable in physical theorising should eo ipso admit of a (presumably unique), well-defined quantification of that "mutability" – and why we ought to identify it with energy. The need for an explicit argument can be seen also from the following thought. What is supposed to be the relevant notion of changeability: variability within a theory's model or variability across different models of that theory? In case of the latter, mutability in the intended sense and energy as it's customarily understood come apart: energy in physics is a quantity defined *within* a model of a theory – not a measure of variability *across* different models. Now consider the other option: to construe variability as variability *within* a given model of the theory. Immediately, one faces intuitive counterexamples. Think of a completely static Newtonian particle universe: should we regard it as an *immaterial* system just because nothing happens? (Such a model is surely empirically wrong – but that doesn't necessarily imply that it doesn't represent a material/physical system.)

(albeit, of course, not uncontroversial) guide to reality claims – an inference to the best explanation – doesn't seem to underwrite such a criterion: in order to judge whether some quantity is real, on that "abductive" line of reasoning, only its explanatory utility matters – *not*, whether we can ascribe it energy.

Moreover, non-gravitational examples cast doubt upon energy ascription as a necessary reality criterion. Consider, for instance, Fermi's degeneracy pressure. The repulsion between the constituents of a system of electrons doesn't involve any energy transfer (or any true force, produced by an exchange of force carriers). No interaction energy exists. Still, the pressure beween the electrons is doubtlessly real (manifest in the existence of, say, neutron stars).

Thus a spacetime realist can affirm the reality of spacetime structure to which gravity is reduced in strength-3 geometric theories. At the same time, she can deny that gravitational energy exists. The tidal deformations that a cloud of dust particles undergoes (e.g. due to a gravitational wave) are real without there being any need to attribute this effect to gravitational energy – gravity's capacity to do work in the sense of deviation from inertial states of motion.

- The second argument for gravitational energy explains the change in (non-gravitational) *matter* energy-stress of systems in the presence of gravity by the *omission* of gravitational energy-stress contributions. That is: In virtue of total energy-conservation, apparent violations in gravitational scenarios indicate the neglect of energy contributions from gravity. The standard presentation of Feynman's sticky-bead argument (Ch. 2) was shown to be an instance of this argument.

  This inference, too, is predicated on a tacit premise – energy conservation. Its local form is: at any point, we can define unambiguously energy-stress 4-fluxes, and they are without sinks/sources. Its global form is: the associated global/integral quantities over suitable regions are well-defined and conserved.

  We saw in our discussion of (C2*) that in generic spacetime theories, this principle can no longer be taken for granted. Its application in the present context requires attention to some subtleties. Local conservation of *non*-gravitational energy-momentum was indeed argued to hold. Apparent violations should, I reasoned, be dismissed as artefacts of unphysical frames: they merit no more realism than fictitious forces in Classical Mechanics. This blocks the appeal to local energy conservation in the preceding argument for gravitational energy: any local violations of (non-gravitational) energy-momentum are artefacts of unphysical descriptions; they aren't the manifestations of neglected gravitational contributions.

In contrast to local energy conservation, global (non-gravitational) energy conservation was found to hold only in special – highly symmetric – spacetimes. Consequently, unless the spacetimes in question belong to this class, increase/decrease of energy-momentum was just a brute fact. In particular, it isn't the manifestation of neglected gravitational contributions. This blocks also the second main argument for gravitational energy.

In short: neither of the two intuitive arguments for the conceptual necessity of gravitational energy holds water. Both rest on substantive background premises that can no longer be taken for granted in modern spacetime and gravitational theories.

Let's now formulate the refined form of (C1):

(C1*) In theories that reduce gravity to inertial structure *local* notions of gravitational energy – i.e. energy fluxes/densities – becomes questionable.

The formal definability of *global* notions, however, becomes contingent on suitable properties of the theory's spacetime structure (asymptotic symmetries). Whether such global notions should be taken seriously – whether they merit a realist stance – depends on the extent to which this prerequisite spacetime structure should be taken seriously (plays some explanatory role).

This refined hypothesis, too, was borne out by the case studies of Ch. II-VI:

- In GR, the paradigmatic strength-3 geometric theory of gravity, gravitational energy becomes problematic both locally and globally.

  We reviewed and dismissed various proposals for local gravitational energy-stress (Ch. III and IV). Pseudotensors, in particular, were rejected as non-geometric objects, defying a natural invariance condition, necessary for representing physical quantities (Ch. IV).

  The considered standard global notions presupposed asymptotically symmetric spacetimes. This was found to be either highly idealised by itself, or an "idle posit" of an idealisation. It was therefore concluded that we have no empirical reason to assume global energy conservation. A promising, unexplored avenue for global notions, however, was briefly alluded to.[182]

[182] Bracketing the prospects of these alternatives, I argued for eliminativism about gravitational energy in GR: we should reject both local and global notions of gravitational energy. This position also has explanatory pay-off for a few issues.
First, eliminativism provides an elegant solution to the puzzle why it's so difficult to localise (to find a genuinely physical, local expression for) gravitational energy: evidently, one can't localise what doesn't exist.
Secondly, Butcher et al. (2010, p. 2) remark: "In spite of these various difficulties [to localise gravitational energy, P.D.], one aspect of this enduring problem stands opposed to conventional wisdom […]: when gravity and matter

These results carry over to f(R) theories of gravity (see also Dürr, ms*), and Brans-Dicke theory (Dürr, ms).[183]

- Newtonian Gravity (Ch. V) in its *non*-strength-3 geometric variants admits of robust, well-defined notions of local and global gravitational energy.

  An exception is Maxwell-Huygens Gravity: given its scarce conceptual resources, it's not clear how even to define gravitational energy. But this result is in a sense expected: energy is a quantity defined relative to inertial structure – but Maxwell-Huygens spacetime doesn't have a sufficiently robust notion of inertial structure.

  In its strength-3 geometric variant, Newton-Cartan theory, one encounters similar problems for local and (non-trivial) global ones (cf. Curiel, ms, sect. 2).

- Nordström Gravity in its strength-3 geometric version of Ch. VI shares with GR the difficulties in defining a local notion of gravitational energy. Global notions can formally be defined in suitable models; in generic models, however, global gravitational energy is no longer well-defined. Due to Nordström Gravity's gross empirical inadequacy (already at the level of solar system tests), the question of realist commitment towards such global notions becomes moot.

- It deserves to be mentioned that (C1*) is a claim specifically about strength-3 geometric theories. Weaker forms of geometrisation needn't affect the definability of local gravitational energy (Dürr, ms, where the case Teleparallel Gravity are discussed).

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interact, the exchange of energy is local! To see this we need look no further than the sticky bead detector: here, the energy exchange is certainly localized in so far as it takes place only within the confines of the detector." Prima facie, it seems paradoxical that on the one hand, gravitational energy defies localizability; on other hand, the alleged energy exchange with matter is localisable. On eliminativism, the paradox evaporates. For eliminativists, only non-gravitational energy really exists. In the absence of spacetime symmetries, its local or global non-conservation is (by the eliminativist's lights) only detectable in matter energy-stress: it's the non-conservation – the "extra" or "missing" bits in the (non-gravitational) energy balance – that realists about gravitational energy (according to the eliminativists: illicitly) reify. The localisability of the "energy-exchange" merely mirrors the fact that (1) non-gravitational energy is localisable and (2) generically, not conserved.
Finally, eliminativism about gravitational energy also sheds light on an oddity of (non-geometrised) Newtonian Gravity that puzzled already Maxwell in 1846: the energy density of a gravitational field is negative (see Synge, 1972, p. 373). Further peculiarities beset the energy of the Newtonian gravitational field. One concerns the multiplicity of formally definable energy fluxes (op.cit., sect. 4). (To make the issue even more bizarre: some of these fluxes yield finite gravitational velocities!) An eliminativist about GR's gravitational energy can shrug off these oddities: they are merely artefacts of reifying gravitational energy, manifesting themselves in the Newtonian limit, too.

[183] The latter is shown to admit of a strength-3 geometrisation that solves some conceptual puzzles of the received "scalar field-cum-curved Riemannian geometry" interpretation. The physical spacetime that absorbs the Brans-Dicke scalar is non-Riemannian: in addition to the Levi-Civita connection, representing inertial structure, it has a torsionful second connection, cf. Faraoni, 2004, Ch. 1.6, responsible for gravitational effects sui generis, such as the Nordtvedt Effect.)

(C1*) remains per se silent on the status of global notions of gravitational energy, *when* it's formally definable. My negative verdict on the explanatory utility – on which my eliminativism of also global gravitational energy *in GR* was based – was expressly tentative (and programmatic). In Ch. V, it was suggested that in this regard progress would be achieved by adverting to the explanatory utility of such notions: a realist interpretation of global notions of gravitational energy, treating them as real physical quantities (rather than merely formal constructions) would be licenced, should they play an important role in explaining salient astrophysical and/or cosmological phenomena (e.g. energy processes in jets of black holes) – a follow-up project that calls for numerous case studies.

VI.2. Outlook

I'll finally outline two lines of future research that emanate from the work presented above – to be tackled in future work. The first concerns a comparison of the problems diagnosed for gravitational energy in GR and those encountered in non-Abelian gauge theories. A second line of inquiry seeks on the one hand, to uncover the conditions of possibility of strength-3 geometrisation; on the other hand, it examines strength-1 and strength-2 geometrisation as putative theoretical virtues.

One natural follow-up will be to scrutinise Deser's (2019) suggestion that GR's problems with defining local and global gravitational energy are strongly analogous to those encountered in non-Abelian Yang-Mills theories, such as chromodynamics (Abbott & Deser, 1982): in the latter, too, well-defined (i.e. gauge-invariant) both locally and globally conserved charges require[184] (i) a split of the gauge field into a background (which constitutes a solution of the source-free field equations) and a (not necessarily small) perturbation around this background, as well as (ii) the existence of Killing fields of this background.

It would be instructive (especially with with an eye to the GR exceptionalism/egalitarianism controversy of Chapter IV) to fully unpack this comparison between GR and non-Abelian Yang-Mills theories. Two questions are of special pertinence.[185] The first concerns the conceptual significance, interpretation and justification of *breaking* local gauge invariance, as per (i) and

[184] The reason for this this, according to Abbot & Deser, lies in the fact that, in contrast to Abelian Yang-Mills theories, the field strengths in non-Abelian ones aren't invariant; rather, they are gauge vectors.
[185] Many thanks to Andrea Ferrari (Durham) for an extended discussion.

(ii).[186] Secondly, one should clarify if (i) and (ii) might actually impose relevant *physical* restrictions. (Vis-à-vis the rich structure of the theory's vacuum solution space – in particular, in light of the fact that *not* all such solutions are gauge-equivalent – singling out one such vacuum solution as a privileged background field has certainly *some* physical consequences.)

Suppose that Deser's claimed analogy between GR and non-Abelian Yang-Mills theories indeed holds. Then, a fundamental question would be whether it – i.e. need for certain additional assumptions of background structure for the definition of currents/charges – holds in virtue of *technical* reasons (a plausible candidate being the theories' non-Abelian character[187]), or whether there might exist a more abstract, general principle. (The results of this dissertation might indeed suggest so: the idea that field currents/charges invariably require a sufficiently robust, fixed background – be it spatiotemporal or some background fields – in order to be meaningfully defined.)

A second set of follow-up questions delves more deeply into geometrisation in the form instantiated in GR, i.e. strength-3 geometrisation – as we saw, a momentous re-conceptualisation of gravity. This dissertation studied its *consequences*. Equally important, however, is to inquire into its presuppositions, i.e. its prerequisite conditions – the facts about gravity that make strength-3 geometrisation possible: due to what feature does gravity admit of a reduction to inertial structure? (A criterion often mentioned in this context, and worthy of analysis, is universal coupling, see e.g. Will, 2018, Ch.3.) This raises a further question: are also other (non-gravitational) quantities amenable to strength-3 geometrisation? That is: Do also other physical quantities display this feature that enable its geometrisation?

In this context, it will be important also to probe the *merits* of strength-3 geometrisation. What justifies its distinction as the "*optimum* case for geometrisation" (Lehmkuhl, 2009, p. 279, my emphasis)? That is: Which theoretical virtues does strength-3 geometric theories exhibit that strength-2 geometric theories lack (or possess only to a lesser degree)?[188]

[186] In GR, this locally broken gauge invariance takes the form of restricting general covariance to diffeomorphisms that reduce to the identity at infinity (so as to preserve asymptotic flatness). In terms of Belot (2008), this brings about a transition from GR's general covariance to "*locally* general covariance".

[187] Recall that the diffeomorphism group, which in some regards may be viewed as taking over the role of GR's gauge group (as emphasised e.g. by Straumann, 2013, p. 6) is non-Abelian.

[188] It's, of course, yet another question how seriously we should take these theoretical virtues (whatever they be). In particular, it's apriori unclear that they are indeed truth-conducive or even genuinely explanatory – a caveat that even applies to unificatory power (cf. Karaca, 2012 for the case of Kaluza-Klein theory).

A particularly relevant virtue, underscored by Einstein himself (Lehmkuhl, 2014), is arguably unificatory power (as explicated e.g. by Kitcher, 1981, 1989; Maudlin, 1996; Bartelborth, 2002). One may hence wonder whether strength-2 geometric theories count as less unificatory than strength-3 geometric ones.

If so, what is it about the latter's reduction to inertial structure that effects this greater unification (or whatever its salient theoretical virtue it's taken to have)? That is: Does it accrue from the *reduction/elimination* simpliciter achieved by strength-3 geometrisation or, more specifically, from the reduction to *inertial structure*?

This is a pressing question: theories are easy to find whose inertial structure is either difficult to identify unambiguously (cf. Menon & Read, 2019 for some examples), or in which the notion's meaningfulness itself is suspect. (Think of quantum gravity scenarios, such as Loop Quantum Gravity, in which there seems to exist no continuous spatiotemporal structure at the most fundamental level of description.). Even in GR, one may baulk at it: Einstein himself admitted, what constitutes inertial structure is imported from pre-general-relativistic theories "to ensure continuity of thought" (see Lehmkuhl, 2014, 2019). What then is so special about inertial (or more generally, kinematic) structure that reduction to it makes it the "optimum case of geometrisation"?[189] Indeed, some authors (such as Brown, 2005) dismiss the very distinction between kinematics and dynamics, whereas others affirm (e.g. DiSalle, 1995; Friedman, 2001, 2002; Curiel, 2018) its fundamentality.

The converse of the question about the privileged status of reduction to inertial structure concerns the status of strength-1 and strength-2 geometrisation. Are they per se, as Einstein seemed to believe, completely without merit? Do they possess any unificatory power? In which sense is their geometrisation weaker than that of strength-3?

To demarcate the three different forms of geometrisation from each other, both regarding their conceptual nature and scientific value, it will be illuminating to apply a more fine-grained distinction between various forms of reduction (see Van Gulick, 2001). On the one hand, we have ontological reduction – an intrinsic relation of items in the world, e.g. objects, properties

[189] Relevant in this regard (but of course in need of elucidation) is that inertial structure usually accorded a distinguished status as encoding natural or default states (cf. Maudlin, 2005, Ch. 5; Hüttemann, 2018, part III): only deviation from it is seen in need for explanation in terms of a cause. What is the basis of this privileged status? Is it perhaps "merely" pragmatics?

or processes – including elimination, identity, composition, and supervenience. On the other hand, we have representational reduction – a relation between representational objects, e.g. concepts, theories or frameworks – including Nagelian derivability or replacability. It's prima facie tempting to identify strength-1 geometrisations with instances of representational reduction; by contrast, the salient difference between strength-2 and strength-3 geometrisation can be cashed out as instances of distinct forms of ontological reduction.

VIII. Bibliography

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